Electrostatics and Fields

Basics:

Field Force - Acts at a distance

- Ex. Gravity, E-Field

Electron Affinity:

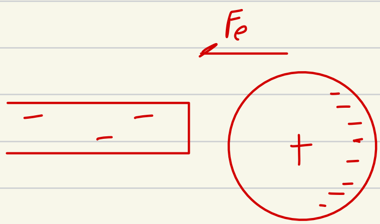
- Object with greater affinity attracts electrons
- Total charge conserved; it can be transferred
- Electrons transferred in “quanta” or packages of charge
- Only electrons can transfer b/c protons held by strong nuclear force in the atom
- Triboelectric Series - Demonstrates electron affinity

Triboelectric Charging:

- Charging via physical rubbing etc.

Polarization:

- Electrons polarized and now the left side remains positive, which is attracted to negative rod
- Polarization can never repel a neutral object

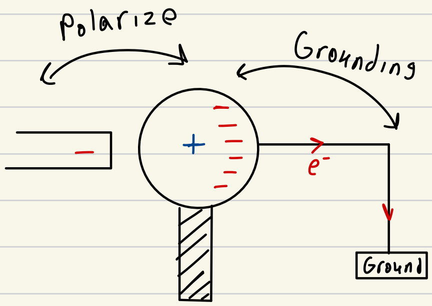


Grounding:

- Acts as an electron giver or taker
- Can neutralize or make object positive by taking extra electrons or turn object negative by giving electrons

Induction:

- Polarize object then ground it



Electric Fields Basics

- Charges create an electrical field similar to how masses cause a gravitational field
  - Size/strength of field depends on size/strength of charge
- Other charges can sense this field
- Objects must both have electric charges similar to how for gravity, both objects must need mass

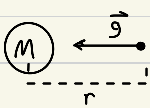
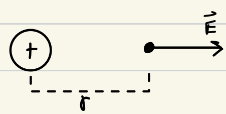
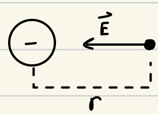
Charge Notation:
- Variable = q/Q
- Measured in Coloumbs (c)

$mC \rightarrow C \times 10^{-3}$

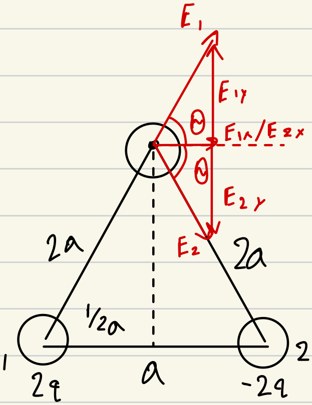
$\mu C \rightarrow C \times 10^{-6}$

$nC \rightarrow C \times 10^{-9}$

Comparison to Gravity:

Gravity	E-Field
m, M	q, Q
	
$\lvert\vec{g}\rvert \propto M$ $\lvert\vec{g}\rvert \propto 1/r^2$	
$\lvert\vec{g}\rvert = \frac{GM}{r^2}$	$\lvert\vec{E}\rvert \propto Q$ $\lvert\vec{E}\rvert \propto 1/r^2$
	$\lvert\vec{E}\rvert = \frac{k\lvert Q\rvert}{r^2}$ $\lvert\vec{F_e}\rvert = \lvert q\rvert\lvert\vec{E}\rvert = \frac{k\lvert Q\rvert\lvert q\rvert}{r^2}$

Example w/ Variables:



$|E_1| = |E_2| = \frac{k\,2q}{4a^2} = \frac{kq}{2a^2}$

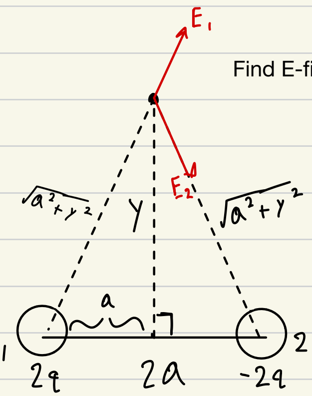
$|E_{1y}| = |E_{2y}|$ ∴ they cancel

$\Sigma E = E_{1x} + E_{2x} = 2E_{1x} = 2E_1 \cos\theta$

↓ $\cos\theta = \frac{1}{2}a / 2a = \frac{1}{4}$

$\Sigma E = (\cancel{2})\frac{kq}{\cancel{2}a^2}\left(\frac{1}{4}\right) = \boxed{\frac{kq}{4a^2} \text{ to right}}$

$\Sigma F_e = q\,\Sigma E = \boxed{kq/4a^2 \text{ to right}}$



Find E-field if test charge was on any point on perpendicular bisector y

$|E_1| = |E_2| = \frac{k\,2q}{(\sqrt{a^2+y^2})^2} = \frac{2kq}{a^2+y^2}$

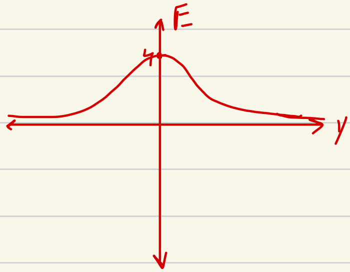
$\Sigma E = E_{1x} + E_{2x}$

↓ $\cos\theta = \frac{a}{\sqrt{a^2+y^2}}$

$\Sigma E = 2E_{1x} = 2\left(\frac{2kq}{a^2+y^2}\right)\left(\frac{a}{\sqrt{a^2+y^2}}\right) = \frac{4kqa}{(a^2+y^2)^{3/2}}$

Make all constants equal to 1 to get an idea on how the graph looks

$E = \frac{4}{(1+y^2)^{3/2}}$



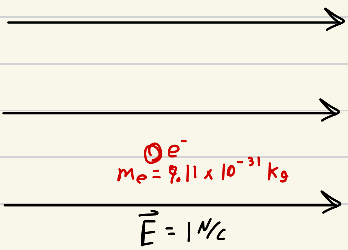
In order to test what happens if y becomes very large:

$\Sigma E = \frac{4kqa}{(y^2(a^2/y^2+1))^{3/2}}$

$\Sigma E = \frac{4kqa}{y^2(a^2/y^2+1)^{3/2}}$

$\lim_{y\to\infty} \Sigma E = \frac{4kqa}{y^2(\cancel{a^2/y^2}+1)} = \frac{4kqa}{y^3}$

The Electric Field Force



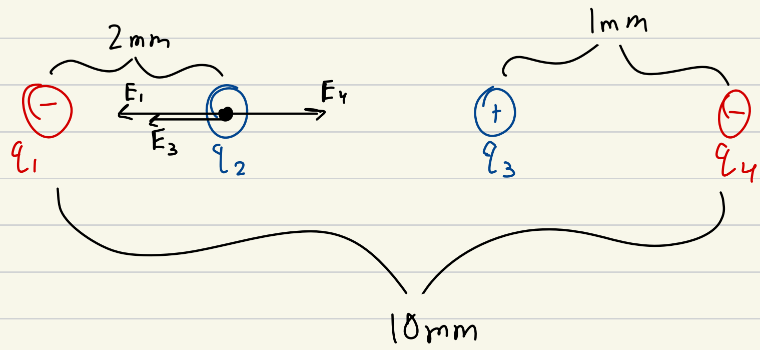
- q = -e = 1.6 x 10^-19 C
- $\vec{E} = E\,\hat{i}$

$$|\vec{F}_e| = |-e|\,|\vec{E}|$$

$$|\Sigma F| = m_e|\vec{a}| = |\vec{F}_e|$$

$$|\vec{a}| = |\Sigma \vec{F}|/m_e = \frac{eE}{m_e}$$

Electric Fields and Forces Superposition 1



Determine electric Field at q2 and net electric force on q2

$q_1 = -4nC \quad q_2 = 6nC \quad q_3 = 8nC \quad q_4 = -5nC$

1) Find forces that effect q2, (*remember q2 doesn't effect itself)
2) Find Magnitude of E-Fields

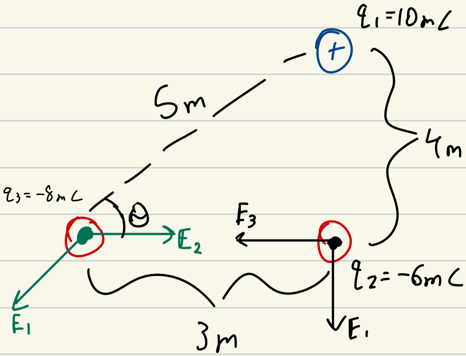
$$|E_1| = \frac{k|q_1|}{r^2} = \frac{9\times10^9(4\times10^{-9})}{.002^2} = 9\times10^6 \text{ N/C} \leftarrow$$

$$|E_3| = \frac{k|q_3|}{r^2} = \frac{9\times10^9(8\times10^{-9})}{.007^2} = 1.47\times10^6 \text{ N/C} \leftarrow$$

$$|E_4| = \frac{k|q_4|}{r^2} = \frac{9\times10^9(5\times10^{-9})}{.008^2} = 7.03\times10^5 \text{ N/C} \rightarrow$$

$$\Sigma E = E_1 + E_3 - E_4 = \boxed{9.77\times10^6 \text{ N/C} \leftarrow}$$

$$\Sigma F_e = |q_2||\Sigma E| = (6\times10^{-9})(9.77\times10^6) = \boxed{.0586 \text{ N} \leftarrow}$$



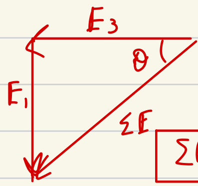
Find net E-Field and net force at location of q2 and q3

$^{*}|E| = \frac{k|q|}{r^2}$, $k = 9\times10^9$

q_1)

$E_1 = \frac{k(10\times10^{-3})}{4^2} = 5.625\times10^6$ N/C

$E_3 = \frac{k(8\times10^{-3})}{3^2} = 8\times10^6$ N/C



$\Sigma E = \sqrt{E_3^2 + E_1^2} = 9.78\times10^6$ N/C

$\theta = \tan^{-1}(E_1/E_3) = 35.1°$ S of W

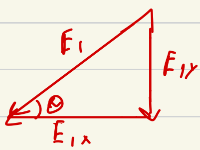
$\boxed{\Sigma E_2 = 9.78\times10^6 \text{ N/C @ } 35.1° \text{ S of W}}$

$\Sigma F_e = q_2|\Sigma E| = \boxed{58677.6 \text{ N @ } 35.1° \text{ N of E}}$ *B/c $q_2 < 0$

q_3)

$E_1 = \frac{k(10\times10^{-3})}{5^2} = 3.6\times10^6$ N/C

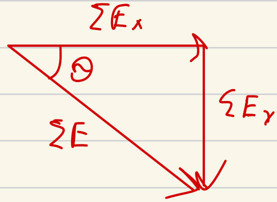
$E_2 = \frac{k(6\times10^{-3})}{3^2} = 6\times10^6$ N/C



$E_{1x} = E_1\cos 53.1° = 2.16\times10^6$ N/C

$E_{1y} = E_1 \sin 53.1° = 2.88\times10^6$ N/C

$\theta = \tan^{-1}(4/3) = 53.1°$



$\Sigma E_x = 6\times10^6 - (2.16\times10^6) = 3.84\times10^6$ N/C

$\Sigma E_y = 2.88\times10^6$ N/C

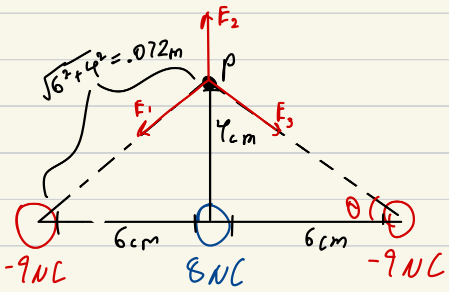
$\Sigma E = \sqrt{\Sigma E_x^2 + \Sigma E_y^2} = \boxed{4.8\times10^6 \text{ N/C @ } 36.9° \text{ S of E}}$

$\theta = \tan^{-1}(\Sigma E_y/\Sigma E_x) = 36.9°$ S of E

$\Sigma F_e = |q_3||\Sigma E| = \boxed{38.4 \text{ N @ } 36.9° \text{ N of W}}$

Made with Goodnotes

Electric Fields and Forces Superposition 2

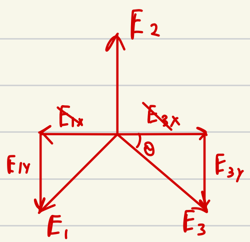


Determine the electric force that would act on electron at P

* $E = \frac{k|q|}{r^2}$, $k = 9\times 10^9$

$E_1 = 9\times10^9(9\times10^{-6})/.072^2 = 1.5625\times10^7\ N/C = E_3$

$E_2 = 9\times10^9(8\times10^{-6})/.04^2 = 4.5\times10^7\ N/C$

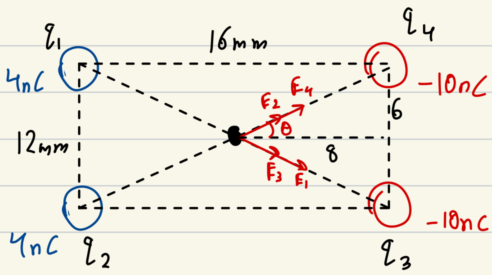


$\theta = \tan^{-1}(4/6) = 33.7°$

$E_{1y} = E_{3y} = E_1 \sin\theta = 1.5625\times10^7 \sin 33.7 = 8.67\times10^6\ N/C$

$\Sigma E = E_2 - 2(E_1) = 2.77\times10^7\ N/C$ up

$\Sigma F_e = |q||\Sigma E| = 1.6\times10^{-19}(2.77\times10^7) = 4.43\times10^{-12}\ N$ down



Find E-Field and force due to E-field at the point

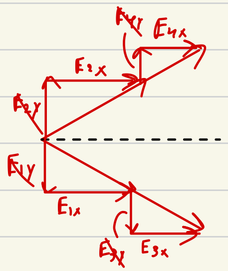
$* \; E = \frac{k|q|}{r^2}, \; k = 9\times10^9$

$E_1 = E_2 = 9\times10^9(4\times10^{-9})/.01^2 = 3.6\times10^5 \text{ N/C}$

$r = \sqrt{8^2+6^2} = .01\text{m}$

$E_3 = E_4 = 9\times10^9(10\times10^{-9})/.01^2 = 9\times10^5 \text{ N/C}$

$\theta = \tan^{-1}(6/8) = 36.9°$



$\Sigma E = E_{1x} + E_{2x} + E_{3x} + E_{4x}$

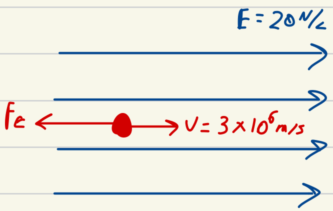
$= 2(3.6\times10^5)\cos 36.9 + 2(9\times10^5)\cos 36.9$

$= 2.02\times10^6 \text{ N/C} \rightarrow$

$\Sigma F_E = |q||\Sigma E| = 3\times10^{-3}(2.02\times10^6) = 6.06\times10^3 \text{ N} \rightarrow$

Electric Forces and Motion:

Electron traveling at 3e6 m/s enters a uniform 20 N/C electric field as shown in figure. How far does electron travel before turning around?



B/c electron is negatively charged, electrical force is pointed in opposite direction

Since force pointed in opposite direction, electron slowly loses speed and turns around

$\Sigma F_e = e|\Sigma E|$

$e\Sigma E = m_e a \rightarrow a = \frac{e\Sigma E}{m_e}$

$e = 1.6 \times 10^{-19}\ N/C$

$E = 20\ N/C$

$m_e = 9.11 \times 10^{-31}\ kg$

$a = \frac{(1.6 \times 10^{-19})(20)}{9.11 \times 10^{-31}} = 3.51 \times 10^{12}\ m/s^2$

$a = -3.51 \times 10^{12}\ m/s^2$

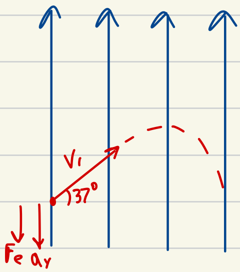
$V_1 = 3 \times 10^6\ m/s$

$V_f = 0\ m/s$

$V_f^2 = V_i^2 + 2a\Delta x$

$\boxed{\Delta x = 1.13\ m}$

Electron moving with velocity of 3x10^6 m/s at 37 degrees with the horizontal enters a uniform 500 N/c vertical electric field. How high does electron rise? How far does electron travel before it comes back to starting height

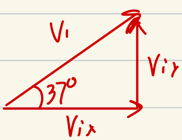


$F_e = e \Sigma E$

$ma = e \Sigma E$

$9.11 \times 10^{-31} a = 1.6 \times 10^{-19} (500)$

$\boxed{a = 8.78 \times 10^{13} \, m/s^2}$



$V_{ix} = V_i \cos 37 = 2.4 \times 10^6 \, m/s$

$V_{iy} = V_i \sin 37 = 1.8 \times 10^6 \, m/s$

Max height:

$V_{fy}^2 = V_{iy}^2 + 2 a_y \Delta x$

$0 = (1.8 \times 10^6)^2 - 2(8.78 \times 10^{13}) \Delta y$

$\boxed{\Delta y = .018 \, m}$

$V_{fy} = V_{iy} + at$

$-1.8 \times 10^6 = 1.8 \times 10^6 - 8.78 \times 10^{13} t$

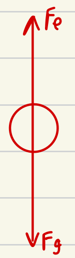
$\boxed{t = 4.1 \times 10^{-8} \, s}$

Made with Goodnotes

Electric Forces and Newtons Laws

A charged 0.5kg ball falls with a constant velocity in a uniform vertical electric field. The charge on the ball is -4nC. Determine the magnitude and direction of the Electric Field.

$\text{Const } \vec{v} \rightarrow a = 0\,m/s^2$



$m = .5kg$

$q = -4 \times 10^{-9} C$

$\Sigma F = 0$

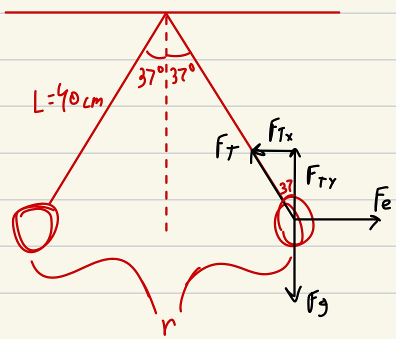
$F_e = F_g$

$q \Sigma E = mg$

$(-4 \times 10^{-9}) \Sigma E = .5(9.8)$

$\boxed{\Sigma E = 1.225 \times 10^{9}\, N/C \text{ South}}$

The charged pendulums of mass 50g each, hang. Determine charge on pendulums and the tension in the string. Assume they carry same charge



$m = .05kg$

$L = .4m$

$T_x = T \sin 37 =$

$T_y = T \cos 37 =$

$\Sigma F_x = 0$

$T_x = F_e$

$\downarrow$

$.61 \sin 37 = 3.91 \times 10^{10} q^2$

$\boxed{q = 3.07 \times 10^{-6} C}$

$\Sigma F_y = 0$

$T_y = F_g$

$\downarrow$

$T \cos 37 = .49$

$\boxed{T = .61N}$

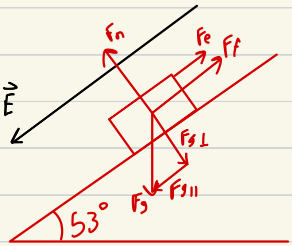
$F_g = mg = .49N$

$F_e = \frac{K q_1 q_2}{r^2} = \frac{K q^2}{r^2}$

$r/2 = L \sin 37 \rightarrow r = .48m$

$\rightarrow F_e = 3.91 \times 10^{10} q^2$

A 20kg box carrying a charge of -2mC slide down a 53 degree ramp, in the presence of a 600 N/c electric field pointing down the incline. Coefficient of friction between box and ramp is 0.2. Determine acceleration of box.



$m = 20\,kg$

$\mu = .2$

$E = 6000\,N/C$

$q = .002\,C$

$F_g = 9.8m = 196\,N$

$F_{g\perp} = F_g \cos 53 = 117.96\,N$

$F_{g\parallel} = F_g \sin 53 = 156.5\,N$

$F_e = qE = 12\,N$

$\Sigma F_\perp = 0$

$F_n = F_{g\perp} = 117.96\,N$

$F_{fk} = \mu F_n$

$F_{fk} = .2(117.96) = 23.6\,N$

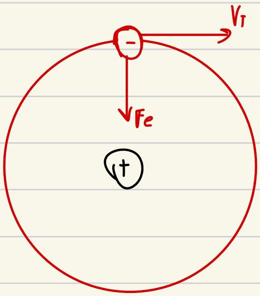
$\Sigma F_\parallel = ma$

$F_{g\parallel} - F_f - F_e = ma$

$120.94 = 20a$

$\boxed{a = 6.05\,m/s^2}$

Hydrogen atom consists of a proton in the nucleus and an electron orbiting nucleus in an orbital radius of .053nm. What is orbital velocity of electron?



$r = .053 \times 10^{-9}\,m$

$m = 9.11 \times 10^{-31}\,kg$

$e = 1.6 \times 10^{-19}\,C$

$\Sigma F_{rad} = F_e$

$\frac{mv^2}{r} = \frac{kee}{r^2}$

$v^2 = ke^2/mr$

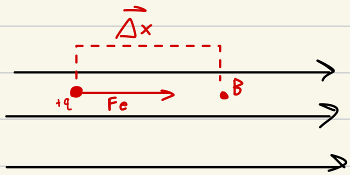
$v^2 = 9\times10^9(1.6\times10^{-19})^2 / 9.11\times10^{-31}(.053\times10^{-9})$

$\boxed{v = 2.18 \times 10^6\,m/s}$

<u>**Electric Potential and Potential Energy**</u>

- When charge moves in electrical field, it experiences electrical force, and work is done.

Consider a charge q moving in uniform electric field from point A to B:

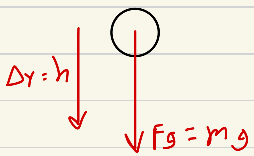


$$F_e = q\vec{E}$$

$$W_e = \vec{F_e} \cdot \vec{\Delta x} = F_e \Delta x \cos\theta = qE\Delta x$$

- In general, the work done by conservative force = -(change in PE)q

Consider falling object of mass m which falls from height h



$$W_g = F_g \Delta y \cos\theta = mgh$$

$$\Delta PE_g = -W_g = -mgh$$

Similarly:

$$\Delta PE_e = -W_e = -qE\Delta x \rightarrow \begin{cases} + \text{ charges lose PE when they move along E field} \\ - \text{ charges gain PE} \end{cases}$$

*Important Note: PE is a scalar quantity, when we calculated changed in PE, we must use sign of the charges.

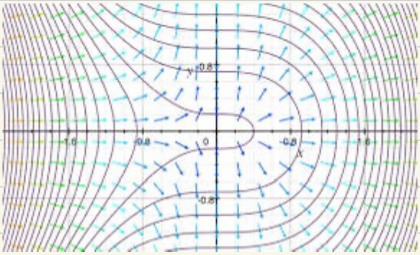
<u>Electrical Potential and Potential Difference</u>

- If a charge q moves from point A to B in an electrical field, and it experiences a change in electrical PE, then we can define a quantity called potential difference (DELTA_V), or change in electrical potential

$$\Delta V = \Delta PE_e / q \quad (J/C) \text{ Joules/Coulomb or (V) volts}$$

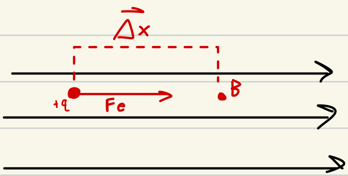
↳ $V_B - V_A$ → At each point in space has a value of V associated with it

- At any region of space, we can draw an equipotential plot where all points on an equipotential line or surface have the same Electric Potential



- Arrows = E-fields
- Curved lines = Equipotentials
- E-Field vectors always perpendicular to equipotentials
- E-Fields always point from regions of higher potential towards regions of lowers potentials

Ex. Uniform E-Fields:



$$\Delta PE_e = -W_e = -qE\Delta x$$

$$\Delta V = \Delta PE_e/q = -E\Delta x \rightarrow E = -\Delta V/\Delta x$$

Ex. Calculate magnitude of electric field in a parallel plate apparatus whose plates are 5mm apart and have potential difference of 300V between them

$\Delta V = 300V$

$\Delta x = 5\times10^{-3}m$

$E = ?$

$\Delta V = -E\Delta x$

$300 = -E(5\times10^{-3})$

$\rightarrow E = 60{,}000\ V/m$

Ex. In a thunder storm, the air must be ionized by a high voltage before a conducting path for a lightning bold can be created. An E-field of about 1,000,000 V/m is required to ionize air/ What would the breakdown voltage in air be if a thundercloud were 3.5km above ground and the electrical field between the cloud and the ground were uniform? You may assume that the cloud and the ground together act like a set of parallel plated

$E = 1000000\ V/m \quad \Delta V = -E\Delta x = -3.5\times10^{9}\ V$

$\Delta x = 3500\ m$

Ex. Two parallel plates are charged up so that potential difference between them is 100V. Find:

a) the PE lost by an electron when it strikes the positive plate if its released from rest at negative plate
b) the speed of the electron (mass 9.11e-31kg) when is strikes plate
c) the PE gained/lost by a proton when it is moved 3mm away from the positive plate, assuming that the full distance between plates is 50mm

a) $\Delta V = 100V,\ q = -1.6\times10^{-19}\ C,\ \Delta V = \Delta PE_e/q \rightarrow 100 = \Delta PE_e/e^- \rightarrow \Delta PE_e = -1.6\times10^{-17}\ J$

b) $V_i = 0\ m/s,\ m_e = 9.11\times10^{-31}\ kg$

$\Sigma W_{nc} = 0$ <- E-forces are conservative

$\cancel{W_{nc}} + \cancel{KE_i} + PE_{e_i} = KE_f + \cancel{PE_{e_f}}$

$PE_i - PE_f = \frac{1}{2}mv_f^2$

$-\Delta PE_e = \frac{1}{2}mv_f^2$

$1.6\times10^{-17} = \frac{1}{2}(9.11\times10^{-31})\,v_f^2$

$\boxed{V_f = 5.92\times10^{6}\ m/s}$

c) $\Delta x = 5\times10^{-2}\ m,\ \Delta V = 100V,\ E = -\Delta V/\Delta x$

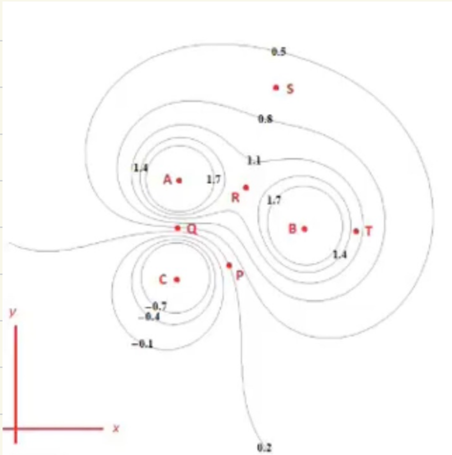
$\therefore E = -100/5\times10^{-2} = -2000\ V/m \rightarrow |E| = 2000\ V/m$

Potential diff between + plate and a point 3mm from it:

$\Delta V = -E\Delta x = -2000\times3\times10^{-3} = -6V$

$\Delta V = \Delta PE_e/q \rightarrow \Delta PE_e \boxed{= -9.6\times10^{-19}\ J}$

$q = -1.16\times10^{-19}\ C$



An electron starts from rest at point P. How fast is it moving when reaching point Q

$V_P = .2V$

$V_Q = .5V$

$\Delta V = .3V$

$\Delta PE_e = q\Delta V$

$\Delta PE_e = -1.6\times10^{-19}(.3)$

$= -4.8\times10^{-20} J$

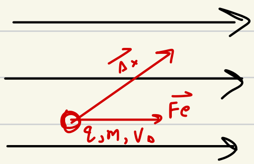
$\cancel{KE_i} + PE_{ei} = KE_f + PE_{ef}$

$-\Delta PE_e = \frac{1}{2} m v_f^2$

$4.8\times10^{-20} = \frac{1}{2}(9.11\times10^{-31}) v_f^2$

$V_f = 3.246\times10^{5}$ m/s

Class notes for Electrical Potential Energy (9/16/24)



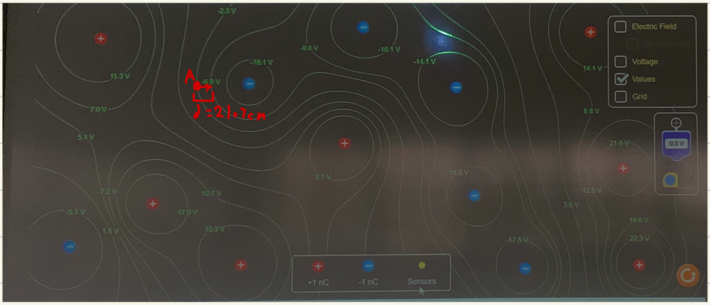
$$W_e = \vec{F_e} \cdot \vec{\Delta x}$$
$$\Delta PE_e = \Delta U_E = -W_e = -F_e|\vec{\Delta x}|\cos\theta = -qE|\vec{\Delta x}|\cos\theta$$

Electric Potential: V

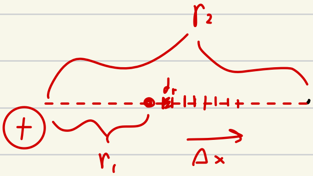
$$\Delta V = \Delta U_E / q$$

- When a particle follows its equipotential line, its change in potential energy stays 0 throughout the line, almost like moving a book at the same height and its gravitational PE stays the same.
- E-Field vectors are ALWAYS perpendicular to equipotential lines

$$\vec{E} = \Delta V / \Delta x$$



$$\Delta V_A = -6.9 - (-2.3) = -4.6\ V$$
$$\Delta x_A = .217m$$
$$|\vec{E}| = -\Delta V / \Delta x = 21.2\ V/m$$



*force is constantly changing so we break displacement into smaller chunks of displacement, d_r

$$F_E = kqQ/r^2$$
$$dW = \frac{kqQ}{r^2} \cdot dr \cos 0°$$
$$dw = kqQ/r^2 \cdot dr$$

$$\int_0^W dw_E = \int_{r_1}^{r_2} \frac{kqQ}{r^2} \cdot dr \rightarrow W_E = kqQ \int_{r_1}^{r_2} \frac{dr}{r^2} = \left[\frac{-kQq}{r}\right]_{r_1}^{r_2}$$

$$W_E = -kQq\,(1/r_2 - 1/r_1)$$

$$= \frac{-kQq}{r_2} + \frac{kQq}{r_1} \rightarrow \Delta U_E = -W_E = \frac{kQq}{r_2} - \frac{kQq}{r_1} \therefore U_E = kQq/r$$

Conservative Forces:

1) When an object moves under influence of a conservative force, the work done by said force is path independent

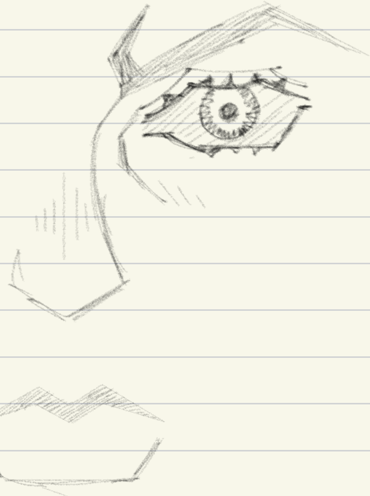
- Ex. Cw depends only on initial and final positions

2) When an object returns to its starting position, Wc must be 0.

3) $\sum W_c + \sum W_{nc} = KE_f - KE_i$

$$-\sum \Delta U_c + \sum W_{nc} = KE_f - KE_i$$

$$\sum U_{ci} - \sum U_{cf} + \sum W_{nc} = KE_f - KE_i$$



U_r

$$F_e = \frac{-\partial U_r}{dr}\hat{r} = -kQq\frac{d}{dr}\left(\frac{1}{r}\right)\hat{r}$$

$$= kQq\left(1/r^2\right)\hat{r}$$

$$= kQq/r^2\,\hat{r}$$

Unit Vector Notes:

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$$

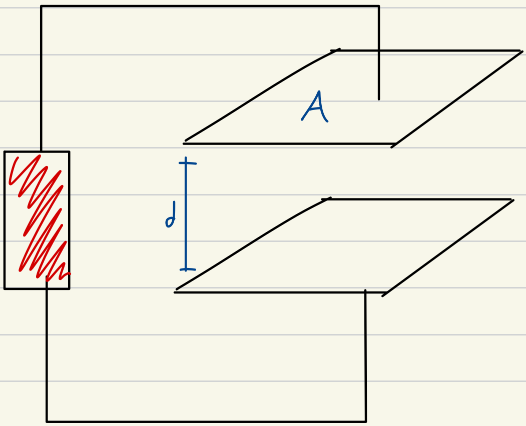
1eV (electron volt)

- 1eV is the Ue of an electron when it passes through potential difference of 1V
- 1.6e-19 in Joules

Capacitors and Circuits

Capacitance (C):

- Amount of charge it can hold



$C = Q/\Delta V$

*Always keeps the same proportion
Measured in Farads (F)

$\vec{E} = \frac{\sigma}{\epsilon_0}$

$\sigma \rightarrow$ Charge/unit area on a plate of capacitor

$\sigma = Q/A$

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

$\vec{E} = \frac{\Delta V_c}{\Delta x} = \frac{\Delta V_c}{d}$

$\frac{\sigma}{\epsilon_0} = \frac{\Delta V_c}{d}$

$\Delta V_c = \frac{\sigma(d)}{\epsilon_0}$

$\Delta V_c = \frac{Qd/A}{\epsilon_0}$

$\Delta V_c = \frac{Qd}{A\epsilon_0}$

$C = Q/\left(Qd/A\epsilon_0\right) = \frac{\epsilon_0 A}{d}$

$C = Q/\Delta V$

$C = \frac{\epsilon_0 A}{d}$

$\vec{E} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$

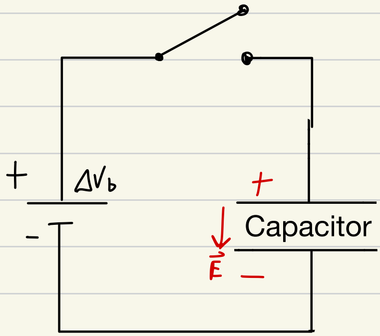
$\Delta V_c = \frac{Qd}{A\epsilon_0}$

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

Alt Formulas:

$U_c = \frac{1}{2} Q V$

$U_c = \frac{1}{2} C \Delta V^2$



__Asymptotic behavior:__

- when the capacitor is initially uncharged, and just begins to charge (immediately after the switch is closed), **the capacitor acts like a bare wire (no resistance)(charge flows freely)**
- After the switch has been closed for a long time, **the capacitor is fully charged and acts like an open switch (no current flows in the circuit).**

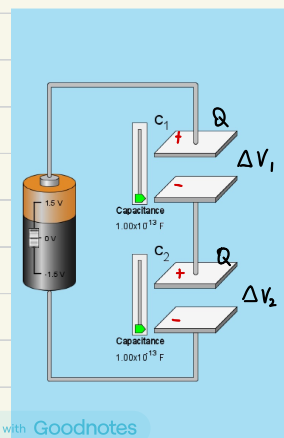
Dielectric:

- An object you put in between the capacitor (ex. Insulator)
- The capacitor polarizes particles in the object causing an opposite E-field
- In other words a dielectric will weaken field of capacitor

$\epsilon \rightarrow$ Permitivity of dielectric

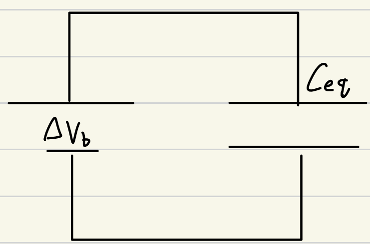
Dielectric const. $k = \frac{\epsilon}{\epsilon_0}$

$C = \epsilon \frac{A}{D}$ || $C = k \epsilon_0 \frac{A}{D}$



$\Delta V_b = \Delta V_1 + \Delta V_2$

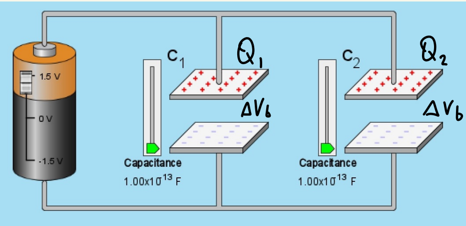
Series $\longrightarrow$



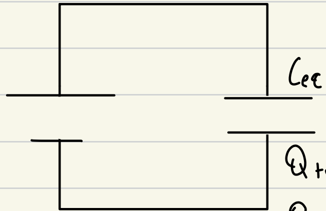
$C_{eq} = \frac{Q}{\Delta V_B} \rightarrow \Delta V_b = \frac{Q}{C_{eq}}$

$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$



Parallel



$Q_{tot} = Q_1 + Q_2$

$Q_{tot} = C_{eq} \Delta V_b$

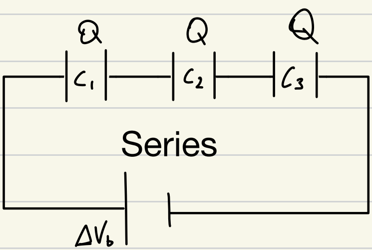
$Q_{tot} = C_1 \Delta V_1 + C_2 \Delta V_2$

$C_{eq} \Delta V_b = \Delta V_b (C_1 + C_2)$

$C_{eq} = C_1 + C_2$

$C_{eq} = \sum_{i=1}^{n} C_i$

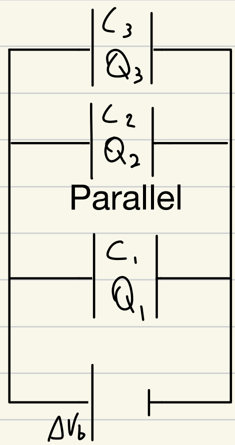
Quick Overview:



$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$

$\Delta V_b = \Delta V_1 + \Delta V_2 + \Delta V_3$

$C = Q/\Delta V$

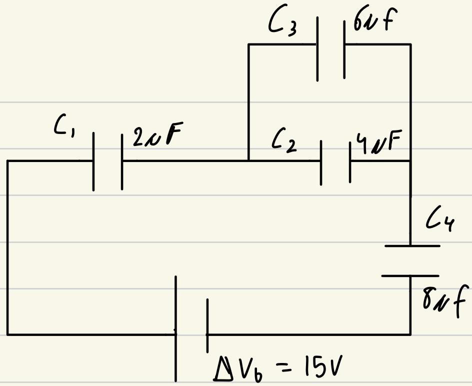


$C_{eq} = C_1 + C_2 + C_3$

$\Delta V_1 = \Delta V_2 = \Delta V_3$

$C = Q/\Delta V$

Ex.



a) Find equivalent capacitance of the combination
b) Find individual potential difference and charge for each capacitor

a) $C_{23} = C_2 + C_3 = 10\mu f$

$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}\right)^{-1} = 1.38\mu f$

b) $Q_{tot} = C_{eq}\Delta V = 20.7\mu C$

$Q_1 = Q_{23} = Q_4 = 20.7\mu C$

$\Delta V_1 = Q_1/C_1 = \frac{20.7}{2} = 10.35V$

$\Delta V_4 = Q_4/C_1 = \frac{20.7}{8} = 2.59V$

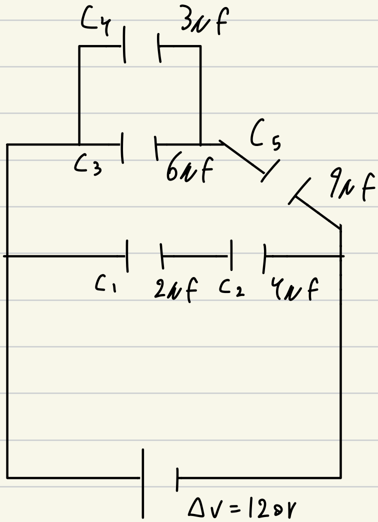
$\Delta V_{23} = Q_{23}/C_{23} = \frac{20.7}{10} = 2.07V$

$\Delta V_2 = \Delta V_3 = \Delta V_{23} = 2.07V$

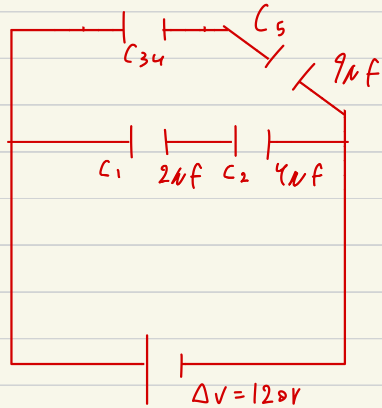
$Q_2 = \Delta V_2 C_2 = 2.07(4) = 8.28\mu C$

$Q_3 = \Delta V_3 C_3 = 2.07(6) = 12.42\mu C$

Ex.

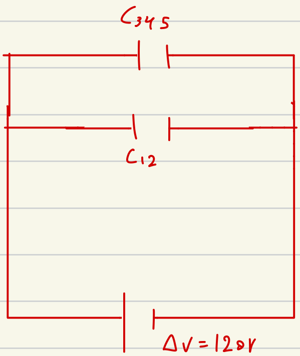


$\equiv$



$C_{34} = C_3 + C_4 = 6 + 3 = 9\mu f$

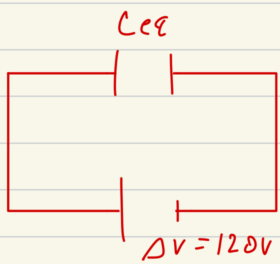
$\equiv$



$C_{345} = \left(\frac{1}{C_{34}} + \frac{1}{C_5}\right)^{-1} = 4.5\mu f$

$C_{12} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = 1.33\mu f$

$\equiv$



$C_{eq} = C_{345} + C_{12} = \underline{5.83\mu f}$

$Q_{tot} = C_{eq}(\Delta V) = \underline{699.6\mu C}$

$\Delta V_{12} = \Delta V_{345} = \underline{120V}$

$Q_{12} = C_{12}\Delta V_{12} = 159.6\mu C$

$Q_{345} = C_{345}\Delta V_{345} = 540\mu C$

$Q_{12} = Q_1 = Q_2 = 159.6\mu C$

$Q_{34} = Q_5 = 540\mu C$

$\Delta V_{34} = \frac{Q_{34}}{C_{34}} = 60V$

$\Delta V_3 = \Delta V_4 = 60V$

$Q_3 = C_3\Delta V_3 = 360\mu C$

$Q_4 = C_4\Delta V_4 = 180\mu C$

$\Delta V_1 = \frac{Q_1}{C_1} = 79.8V$

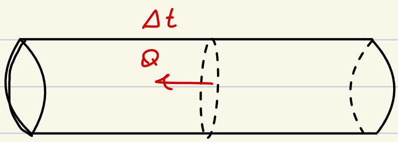
$\Delta V_2 = \frac{Q_2}{C_2} = 39.9V$

$\Delta V_5 = \frac{Q_5}{C_5} = 60V$

More Circuits (Resistors and Current):

Current (I):

- Defined as the rate of flow of charge past a cross section of a wire



$$I = \frac{\Delta Q}{\Delta t}$$

$$\lim_{\Delta t \to 0} I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

$I \propto \frac{1}{R}$

$I \propto \Delta V$

$\therefore I = \Delta V / R$

$\Delta x = v_d \Delta t$

$V = A\Delta x = A v_d \Delta t$

$\rightarrow V = \text{Volume}$

$Q = q_E n V$

n = Electron per unit volume

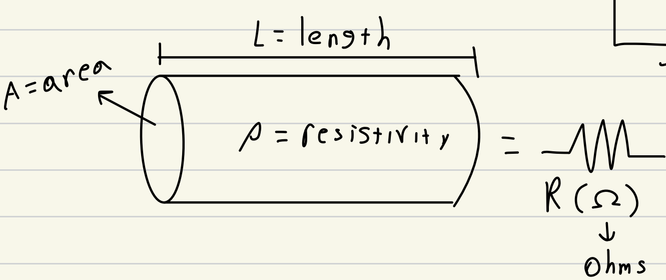
A = Cross section Area

$Q = q_E n A v_d \Delta t$

$I = Q/\Delta t = n q_E v_d A$

- The battery:
  - Provides/Gives potential energy to the charges as they move through

The Resistor:



$I \propto A$

$I = \frac{\Delta V}{R} \quad \therefore R \propto \frac{1}{A}, \quad R \propto \ell$

$$R = \rho \frac{\ell}{A}$$

ρ = resistivity

Conductivity $\rightarrow \sigma = 1/\rho$

- Electrons dont just smoothly pass through a wire
- The electrons collide with the electrons in the copper atom (held on the outside), and have tons of collisions with them, ‘zig-zagging’ their way through the wire
- The **Drift Velocity** (mm/s) is the the average vector the electrons follow

Electric Fields

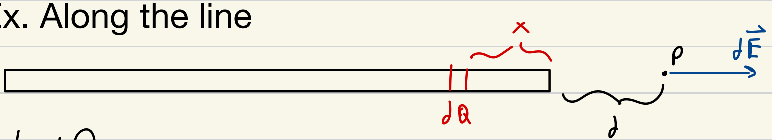
Electric Field Due to a Point Charge:

$$\vec{E} = \frac{kQ}{r^2}$$

Electric field due to a continuous amount of point charge:

Line of finite charge:

Ex. Along the line



$L, +Q$

- Positive dQ causes an electric field dE

$$|d\vec{E}| = \frac{k\,dQ}{(d+x)^2}$$

$\lambda = \frac{Q}{L}$ → Linear charge distribution (uniform)

$$dQ = \lambda\,dx$$

$$\int dE = \int_0^L \frac{k\lambda\,dx}{(d+x)^2}$$

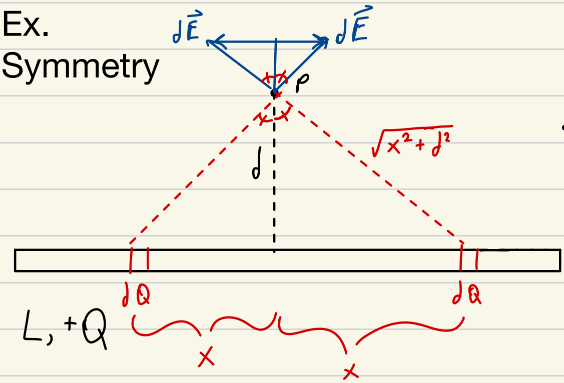
$$E = k\lambda \int_0^L \frac{dx}{(d+x)^2}$$

$$u = d+x$$
$$du = dx$$

$$E = k\lambda \int_0^L \frac{du}{u^2} = \left[k\lambda \cdot -\frac{1}{u}\right]_0^L = \left[-k\lambda \frac{1}{d+x}\right]_0^L = -k\lambda\frac{1}{d+L} + k\lambda\frac{1}{d} = \frac{kQ}{d(d+L)}$$

$$\rightarrow \lambda = \frac{Q}{L}$$

Ex.
Symmetry



x-components equal & opposite
∴ cancel out

$$\lambda = \frac{Q}{L}$$

$$|d\vec{E}| = 2dE_y$$

$$dE = 2dE\cos\theta$$

$$dE = \frac{2k\,dQ}{d+x^2}\,\frac{d}{\sqrt{d^2+x^2}}$$

$$\int dE = \int_0^{L/2} \frac{2k\lambda d\,dx}{(d^2+x^2)^{3/2}}$$

$$E = 2k\lambda d \int_0^{L/2} \frac{dx}{(d^2+x^2)^{3/2}}$$

$$\frac{x}{d} = \tan\theta$$
$$x = d\tan\theta$$
$$dx = d\sec^2\theta\,d\theta$$

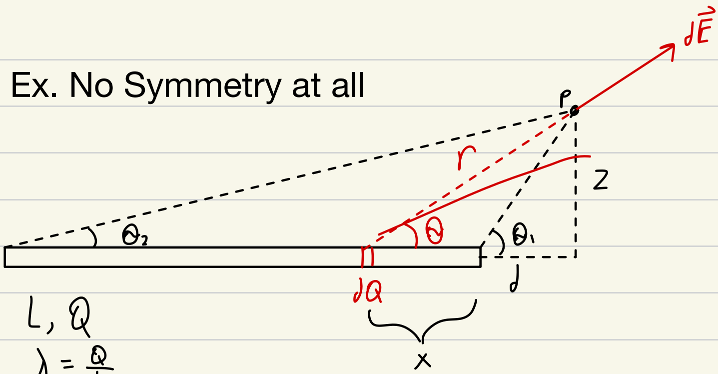
$$d^2+x^2 = d^2+d^2\tan^2\theta$$
$$= d^2(1+\tan^2\theta)$$
$$= d^2\sec^2\theta$$

$$E = 2k\lambda d \int \frac{d\sec^2\theta\,d\theta}{(d^2\sec^2\theta)^{3/2}} = 2k\lambda d^2 \int \frac{\sec^2\theta\,d\theta}{d^3\sec^3\theta} = 2k\lambda d^{-1}\int \frac{1}{\sec\theta}\,d\theta$$

$$E = \frac{2k\lambda}{d}\int\cos\theta\,d\theta = \left[\frac{2k\lambda}{d}\sin\theta\right] = \left[\frac{2k\lambda}{d}\frac{x}{\sqrt{x^2+d^2}}\right]_0^{L/2}$$

$$E = \frac{kQ}{d\sqrt{\frac{L^2}{4}+d^2}}$$

Ex. No Symmetry at all



L, Q

$\lambda = \frac{Q}{L}$

$dQ = \lambda\, dx$

$r = \sqrt{(x+d)^2 + z^2}$

$r = \frac{z}{\sin\theta}$

$$|dE| = \frac{k\,dq}{r^2} = \frac{k\lambda\,dx}{(x+d)^2 + z^2}$$

$$dE_x = \frac{k\lambda\,dx}{(x+d)^2 + z^2}\cos\theta = \frac{k\lambda\,dx}{z^2\csc^2\theta}\cos\theta$$

$$dE_y = \frac{k\lambda\,dx}{(x+d)^2 + z^2}\sin\theta = \frac{k\lambda\,dx}{z^2\csc^2\theta}\sin\theta$$

$$d + x = z\cot\theta$$

$$dx = z(-\csc^2\theta)\,d\theta$$

$$\therefore \quad dE_x = \frac{-k\lambda z\cancel{\csc^2\theta}\,d\theta}{z^2\cancel{\csc^2\theta}}\cos\theta = \frac{-k\lambda\cos\theta}{z}\,d\theta$$

$$dE_y = \frac{-k\lambda z\cancel{\csc^2\theta}\,d\theta}{z^2\cancel{\csc^2\theta}}\sin\theta = \frac{-k\lambda\sin\theta}{z}\,d\theta$$

$$\int dE_x = \int_{\theta_1}^{\theta_2} \frac{-k\lambda\cos\theta}{z}\,d\theta$$

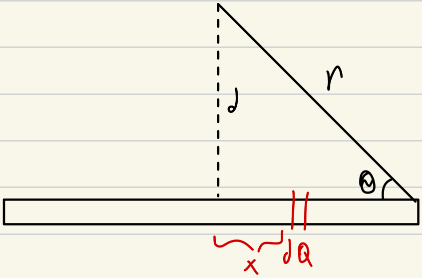
$$E_x = \left[\frac{-k\lambda}{z}\sin\theta\right]_{\theta_1}^{\theta_2}$$

$$E_x = \frac{k\lambda}{z}(\sin\theta_1 - \sin\theta_2)$$

$$\int dE_y = \int_{\theta_1}^{\theta_2} \frac{-k\lambda\sin\theta}{z}\,d\theta$$

$$E_y = \frac{k\lambda}{z}(\cos\theta_2 - \cos\theta_1)$$

Electric Potential:



$$dQ = \lambda dx$$
$$\lambda = QL^{-1}$$

$$dV = \frac{k\,dQ}{r} = \frac{k\lambda\,dx}{\sqrt{d^2+x^2}}$$

$$V = 2\int_0^{L/2} \frac{k\lambda\,dx}{\sqrt{d^2+x^2}} = 2k\lambda \int_0^{L/2} \frac{dx}{\sqrt{d^2+x^2}}$$

$$r = \frac{d}{\sin\theta}, \quad \frac{x}{d} = \cot\theta$$
$$dx = -d\csc^2\theta\, d\theta$$

$$V = 2\int_{\pi-\theta}^{\pi/2} \frac{-k\lambda\csc^2\theta\, d\theta}{\csc\theta}$$

$$V = -2k\lambda \int_{\pi-\theta}^{\pi/2} \csc\theta\, d\theta$$

$$V = -2k\lambda \int_{\pi-\theta}^{\pi/2} \frac{\csc\theta(\csc\theta+\tan\theta)}{(\csc\theta+\tan\theta)}\, d\theta$$

$$u = \csc\theta + \cot\theta$$
$$du = -(\csc\theta\cot\theta - \csc^2\theta)\, d\theta$$

$$V = 2k\lambda \int_{\pi-\theta}^{\pi/2} \frac{du}{u}$$

$$V = \Big[2k\lambda \ln(\csc\theta + \cot\theta)\Big]_{\pi-\theta}^{\pi/2}$$

$$V = -2k\lambda \ln(\csc\theta - \cot\theta)$$

$$V = -2k\lambda \ln\left(\frac{1}{d}\sqrt{\frac{L^2}{4}+d^2} - \frac{L}{2}\,\frac{1}{d}\right)$$

$$V = -2k\lambda \ln\left(\frac{\sqrt{\frac{L^2}{4}+d^2} - \frac{L}{2}}{d}\right)$$

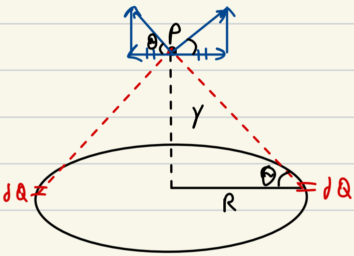
Conversions:

$$\vec{F}_c = -\vec{\nabla} U$$

$$E = -\vec{\nabla} V \therefore V = -\int \vec{E}\, d\vec{r}$$

$$\therefore E = \frac{-dV}{dd}$$

Ex. Ring of Charge



*plausible to pick two diametrically opposite charges

$$dE = \frac{k\,dQ}{r^2} = \frac{k\,dQ}{R^2 + r^2}$$

$$\Sigma E_x = 0$$

$$\Sigma dE = 2\,dE_y = 2\,dE\sin\theta = \frac{2\,dE\,y}{\sqrt{R^2+y^2}} = \frac{2k\,dQ\,y}{(R^2+y^2)^{3/2}}$$

$$E = \frac{2ky}{(R^2+y^2)^{3/2}} \int_0^{Q/2} dQ$$

*use q/2 b/c we are doing half ring (diametrically opposite)

$$\vec{E} = \frac{kQy}{(R^2+y^2)^{3/2}}$$

$$dV = \frac{k\,dQ}{r} = \frac{k\,dQ}{\sqrt{R^2+y^2}}$$

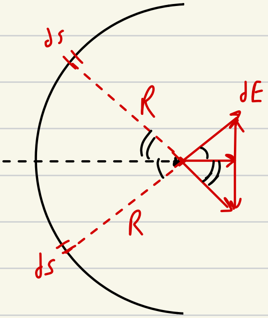
$$\Sigma dV = 2\frac{k\,dQ}{\sqrt{R^2+y^2}}$$

$$V = \frac{2k}{\sqrt{R^2+y^2}} \int_0^{Q/2} dQ$$

$$V = \frac{\cancel{2}k}{\sqrt{R^2+y^2}} \frac{Q}{\cancel{2}}$$

$$V = \frac{kQ}{\sqrt{R^2+y^2}}$$

Ex. Semi Circle of charge



$$\sum dE_y = 0$$

$$\lambda = \frac{Q}{\pi R} = \frac{dQ}{ds}$$

$$dQ = ds\,\lambda$$

$$ds = R\,d\theta$$

$$dQ = R\,d\theta\,\lambda$$

$$dE = \frac{k\,dQ}{R^2} = \frac{kR\,d\theta\,\lambda}{R^2} = \frac{k\,d\theta\,\lambda}{R}$$

$$\sum dE = 2\,dE\cos\theta$$

$$\sum dE = 2\frac{k\,d\theta\,\lambda}{R}\cos\theta$$

$$\sum E = \frac{2k\lambda}{R}\int_0^{\pi/2}\cos\theta\,d\theta$$

$$\sum E = \left[\frac{2k\lambda}{R}\sin\theta\right]_0^{\pi/2}$$

$$\sum E = \frac{2k\lambda}{R}$$

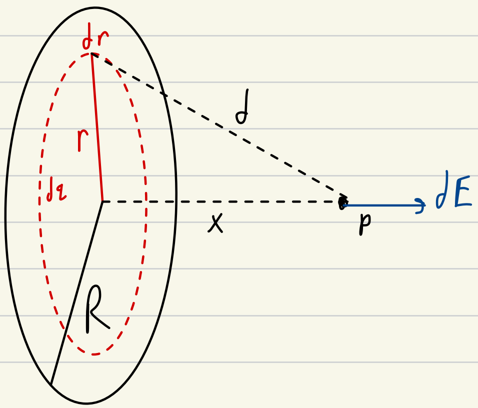
$$dV = \frac{k\lambda\,ds}{R}$$

$$V = \int\frac{k\lambda\,ds}{R}$$

$$V = \frac{\pi R k\lambda}{R}$$

$$V = k\pi\lambda$$

Ex. Disk of Charge



$$\sigma = \frac{Q}{\pi R^2}$$

$$dq = \sigma\, dA$$
$$dq = \sigma\, 2\pi r\, dr$$

$$dE = \frac{k\, dq\, x}{(r^2+x^2)^{3/2}}$$

$$dE = \frac{k x \sigma\, 2\pi r\, dr}{(r^2+x^2)^{3/2}}$$

$$E = 2\pi k x \sigma \int_0^R \frac{r}{(r^2+x^2)^{3/2}}\, dr$$

$$u = r^2 + x^2$$
$$du = 2r\, dr$$
$$dr = du/2r$$

$$E = 2\pi k x \sigma \int_0^R \frac{\cancel{r}}{(u)^{3/2}} \frac{du}{2\cancel{r}}$$

$$E = \pi k x \sigma \int_0^R \frac{du}{u^{3/2}}$$

$$E = \left[\pi k x \sigma \left(-2 \frac{1}{\sqrt{r^2+x^2}}\right)\right]_0^R$$

$$E = \frac{-2\pi k x \sigma}{\sqrt{R^2+x}} + \frac{2\pi k x \sigma}{x}$$

$$E = \frac{2Qk}{R^2}\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)$$

$$dV = \frac{k\, dq}{\sqrt{r^2+x^2}}$$

$$dV = \frac{k 2\pi r \sigma\, dr}{\sqrt{r^2+x^2}}$$

$$u = r^2 + x^2$$
$$du = 2r\, dr$$
$$dr = \frac{du}{2r}$$

$$V = k 2\pi \sigma \int_0^R \frac{r\, dr}{\sqrt{r^2+x^2}}$$

$$V = k 2\pi \sigma \int \frac{\cancel{r}}{\sqrt{u}} \frac{du}{2\cancel{r}}$$

$$V = k\pi\sigma \int \frac{du}{\sqrt{u}} \Rightarrow V = \left[2k\pi\sigma\sqrt{r^2+x^2}\right]_0^R = 2k\pi\sigma\left(\sqrt{R^2+x^2} - x\right) = \frac{2kQ}{R^2}\left(\sqrt{R^2+x^2} - x\right)$$

Gauss's Law

$$\Phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

For complex areas

$$d\Phi_B = \vec{B} \cdot d\vec{A}$$

$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$

For closed surface:

$$\Phi_B = \oiint \vec{B} \cdot d\vec{A}$$

For Electrical Field:

$$\Phi_E = \vec{E} \cdot \vec{A}$$

$$\Phi_E = \iint \vec{E} \cdot d\vec{A}$$

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A}$$

Gauss's Law:

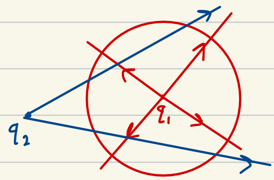
Electric flux passing through any closed surface equals the charge of that closed surface divided by electrostatic constant

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

*For Gravity

$$\Phi_G = \oiint \vec{g} \cdot d\vec{A} = -4\pi G M_{enc}$$

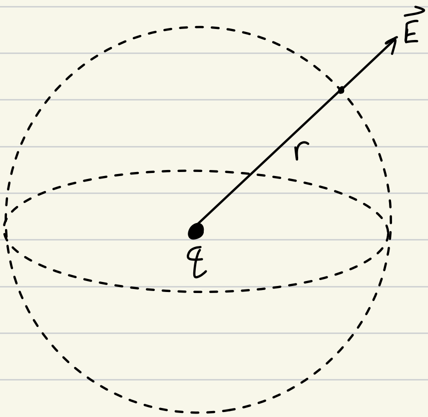
q2 E-field lines enter and exit the surface therefor those cancel



Made with Goodnotes

Gaussian Surface Ex.

- Find E-field using flux. We can make an imaginary surface to calculate
- We want E to be perpendicular to surface so cosine becomes one
- We also want E to be constant everywhere
- Sphere meets this quota



$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \oiint E\,dA$$

$$= E \oiint dA = E A_{surface}$$

$$= E \cdot 4\pi r^2$$

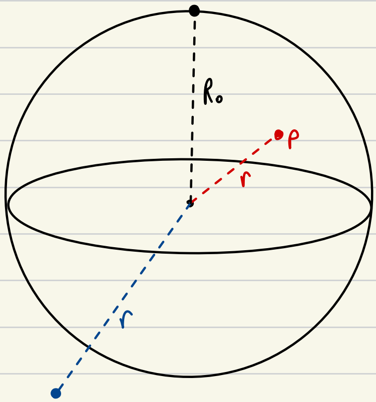
$$E \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2} = \frac{kQ}{r^2}$$

Gaussian Surface should pass through the point
The angle between the E field and surface should be zero or 90
E-field should be constant over surface

$$* \ \Delta V = -\underbrace{\int \vec{E} \cdot d\vec{r}}_{\text{Dot-product}}$$

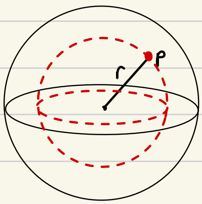
Uniformly Charged Sphere Ex.



$+Q, R_0$

Find
E for $r<R_0$, $R=r_0$, $r>R_0$

$r<R_0$; Use sphere within a sphere as Gaussian surface



$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = \oiint E\,dA$$
$$= E\,4\pi r^2$$

$$\frac{Q_{enc}}{Q} = \frac{\cancel{\frac{4}{3}\pi} r^3}{\cancel{\frac{4}{3}\pi} R_0^3}$$

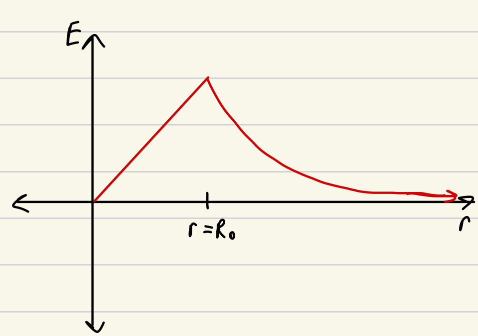
$$Q_{enc} = Qr^3/R_0^3$$

$$4\pi r^2 E = \frac{Q_{enc}}{\epsilon_0}$$
$$4\pi r^2 E = \frac{Qr^3}{\epsilon_0 R_0^3}$$
$$E = \frac{Qr}{4\pi\epsilon_0 R_0^3}$$

$r>R_0$; Same as before but larger sphere

$$\Phi_E = E\,4\pi r^2$$
$$Q_{enc} = Q$$

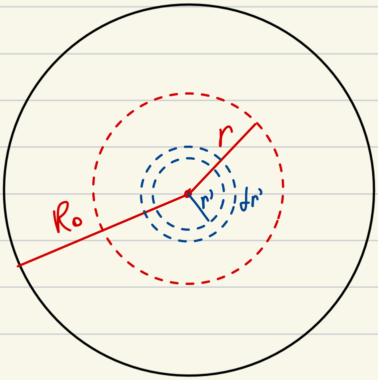
$$E\,4\pi r^2 = \frac{Q}{\epsilon_0}$$
$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$



$r = R_0$

$$E\,4\pi R_0^2 = \frac{Q}{\epsilon_0} \quad \therefore \quad E = \frac{Q}{4\pi\epsilon_0 R_0^2}$$

Insulating Sphere Ex.



$$\rho(r) = Cr^4$$

$$\Phi_E = \oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

Charge volume density changes due to changing radius so we need to break up into increments

$$V = \frac{4}{3}\pi r^3$$
$$dV = 4\pi r'^2 dr'$$

$$dq = \rho(r')\,dV$$
$$\therefore dq = \rho(r')\,4\pi r'^2\,dr'$$

$r < R_0)$

$$q_{enc} = \int_0^r dq$$

$$q_{enc} = \int_0^r \rho(r')\,4\pi r'^2\,dr'$$

$$= \int_0^r Cr'^4\,4\pi r'^2\,dr'$$

$$= \frac{4\pi C r^7}{7}$$

$$E\,4\pi r^2 = \frac{4\pi C r^7}{7\epsilon_0}$$

$$E = \frac{Cr^5}{7\epsilon_0}$$

$r = R_0)$

$$E = \frac{CR_0^7}{7\epsilon_0 R_0^2}$$

$$E = \frac{CR_0^5}{7\epsilon_0}$$

$r > R_0)$

$$q_{enc} = \frac{4\pi C R_0^7}{7}$$

$$E\,4\pi r^2 = \frac{4\pi C R_0^7}{7\epsilon_0}$$

$$E = \frac{CR_0^7}{7\epsilon_0 r^2}$$

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r} = -\int_{\infty}^{r} E\, dr$$

You have to integrate from where POTENTIAL is zero, which would be at infinity

$$E(r) = \begin{cases} \dfrac{Cr^5}{7\epsilon_0}, & r < R_0 \\ \dfrac{CR_0^5}{7\epsilon_0}, & r = R_0 \\ \dfrac{CR_0^7}{7\epsilon_0 r^2}, & r > R_0 \end{cases}$$

$r > R_0)$

$$V = -\int_{\infty}^{r} \frac{CR_0^7}{7\epsilon_0 r^2}\, dr$$

$$V(r) = \frac{CR_0^7}{7\epsilon_0 r}$$

$r < R_0)$

$$V = -\int_{\infty}^{R_0} \vec{E} \cdot d\vec{r} + -\int_{R_0}^{r} \vec{E} \cdot d\vec{r}$$

$$V = -\int_{\infty}^{R_0} \frac{CR_0^7}{7\epsilon_0 r^2}\, dr - \int_{R_0}^{r} \frac{Cr^5}{7\epsilon_0}\, dr$$

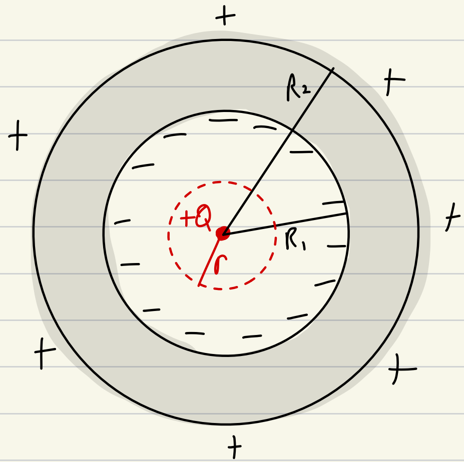
$$V = \frac{CR_0^6}{7\epsilon_0} - \frac{C}{7\epsilon_0}\left[\frac{r^6}{6}\right]_{R_0}^{r}$$

$$V(r) = \frac{CR_0^6}{7\epsilon_0} + \frac{C}{42\epsilon_0}\left(R_0^6 - r^6\right)$$

Made with Goodnotes

Conducting Sphere:

- Surface is an equipotential
- Net field inside the conductor is zero since particles can freely move
- There is E field on the surface though and that E-field is orthogonal to surface



$r < R_1)$

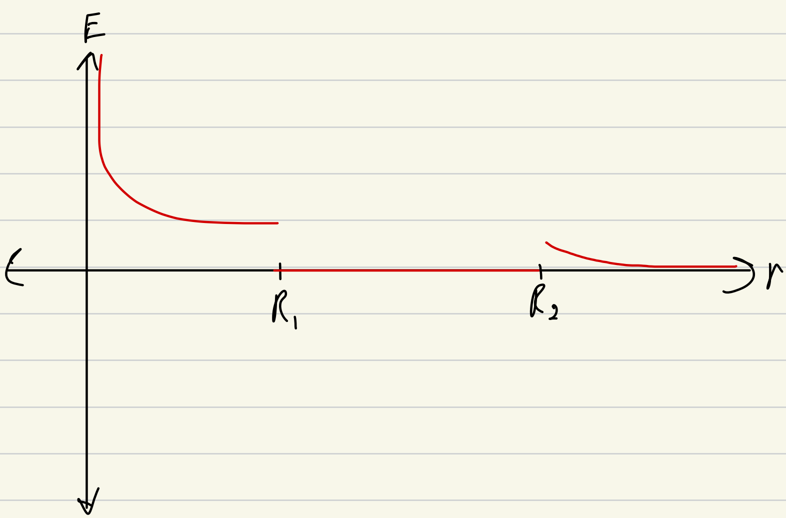
$$E\,4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

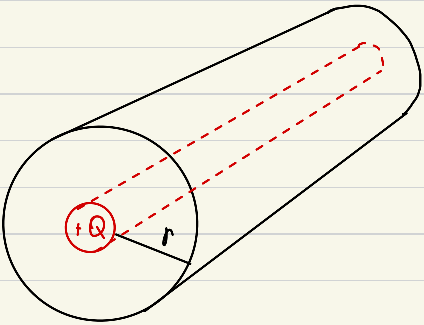
$R_1 < r < R_2)$

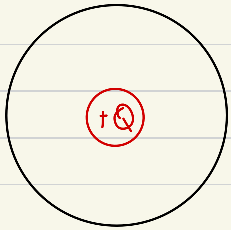
$E = 0$

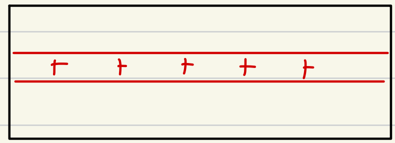
Because this is located in the body of the conductor



Line of Charge Ex.







Gaussian Surface = Cylinder

Whichever point your trying to find E-field of, make a cylinder about that point where the distance from the rod to the point is radius of cylinder

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\underbrace{\cancel{2 \oiint \vec{E} \cdot d\vec{A}}}_{\substack{\text{two end caps} \\ = 0 \text{ b/c} \\ \vec{A} \parallel \vec{E}}} + \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\therefore E \oiint dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E 2\pi r h = \frac{Q_{enc}}{\epsilon_0}$$

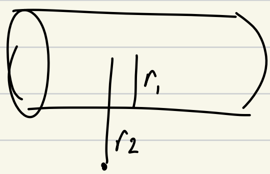
$$Q_{enc} = \lambda h$$

The amount of rod enclosed in the cylinder is simply the cylinders height

$$E 2\pi r h = \frac{\lambda h}{\epsilon}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

ΔV from r_1 to r_2



$$\Delta V = -\int_{r_1}^{r_2} \vec{E} \cdot d\vec{r} = \int_{r_2}^{r_1} \frac{\lambda}{2\pi\epsilon_0 r} dr$$

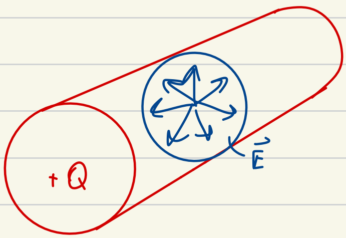
$E \parallel d\vec{r}$

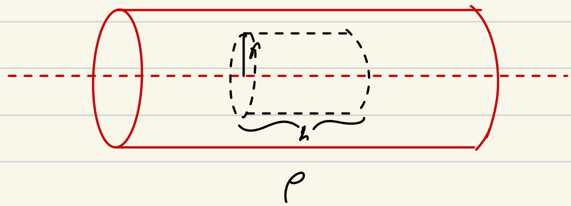
$\cos\theta = 1$

$$= \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

Made with Goodnotes

Infinite Cylinder of Charge Ex.





$r<a)$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E2\pi r\cancel{h} = \frac{\rho\pi r^2\cancel{h}}{\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0}$$

$r=a)$

$$E2\pi a\cancel{h} = \frac{\rho\pi a^2\cancel{h}}{\epsilon_0}$$

$$E = \frac{\rho a}{2\epsilon_0}$$

$r>a)$

$$E2\cancel{\pi}r\cancel{h} = \frac{\rho\cancel{\pi}a^2\cancel{h}}{\epsilon_0}$$

$$E = \frac{\rho a^2}{2r\epsilon_0}$$

Using a function for Charge Density:

$$\rho = Cr$$

$r<a)$

$$E2\pi rh = \frac{Q_{enc}}{\epsilon_0}$$

$$dV = 2\pi rh\,dr$$

$$dQ = \rho\,dv$$

$$dQ = Cr' 2\pi r' h\,dr$$

$$dQ = 2\pi Cr'^2 h\,dr$$

$$Q = 2\pi Ch\int_0^r r'^2\,dr$$

$$Q = \frac{2\pi Ch}{3}r^3$$

$$\therefore\ E\cancel{2\pi}r\cancel{h} = \frac{\cancel{2\pi}C\cancel{h}r^3}{3\epsilon_0}$$

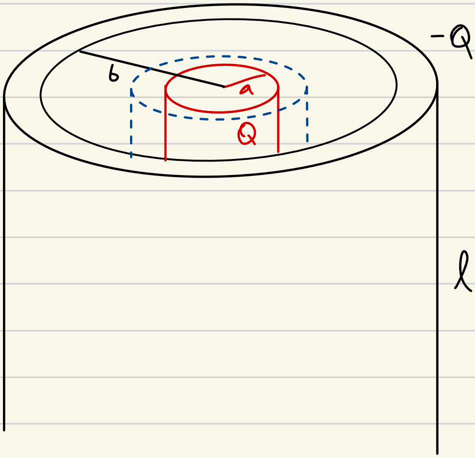
$$E = \frac{Cr^2}{3\epsilon_0}$$

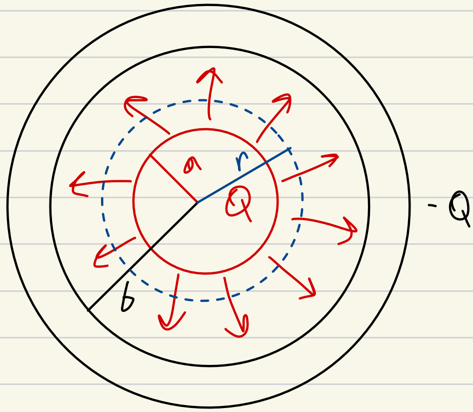
Guass's Law if Dielectric is used and Not Free Space:

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon} = \frac{Q_{enc}}{\underbrace{k}_{\text{Dielectric const.}}\epsilon_0}$$

$$\vec{E} \text{ in dielectric} = \frac{E_0}{K}$$

Cylindrical Capacitor:





Both the outer hollow cylinder and inside cylinder are conductors and therefor field inside them is zero

There is only a field in between the cylinders, so we make the Gaussian cylinder in between there

$a < r < b)$

$$E \oiint d\vec{A} = E 2\pi r h$$

$$\lambda = \frac{Q}{\ell}$$

$$Q_{enc} = \lambda h$$

$$E 2\pi r \cancel{h} = \frac{Qk}{L\epsilon_0}$$

$$E = \frac{Q}{2\pi L \epsilon_0 r}$$

- if dielectric is added, simply add the k term
- If dielectric is only there for certain parts; piece-wise integral

---

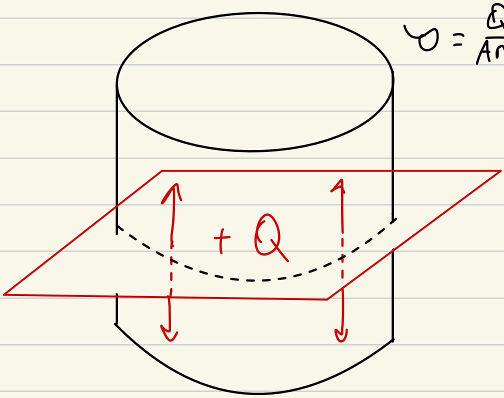
$$C = \frac{Q}{\Delta V}$$

$$= \frac{2\pi \epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

$$\Delta V = -\int_b^a \vec{E} \cdot d\vec{r} = -\int_b^a \frac{Q}{2\pi L \epsilon_0 r} dr = \left[\frac{-Q \ln(r)}{2\pi L \epsilon_0} \right]_b^a$$

$$= \frac{Q}{2\pi \epsilon_0 L} \ln\left(\frac{b}{a}\right)$$

Infinite Sheet of Charge/Pill Box example)



$$\sigma = \frac{Q}{\text{Area}}$$

$$E \oint d\vec{A} = \underbrace{2EA}_{\text{two endcaps}}$$

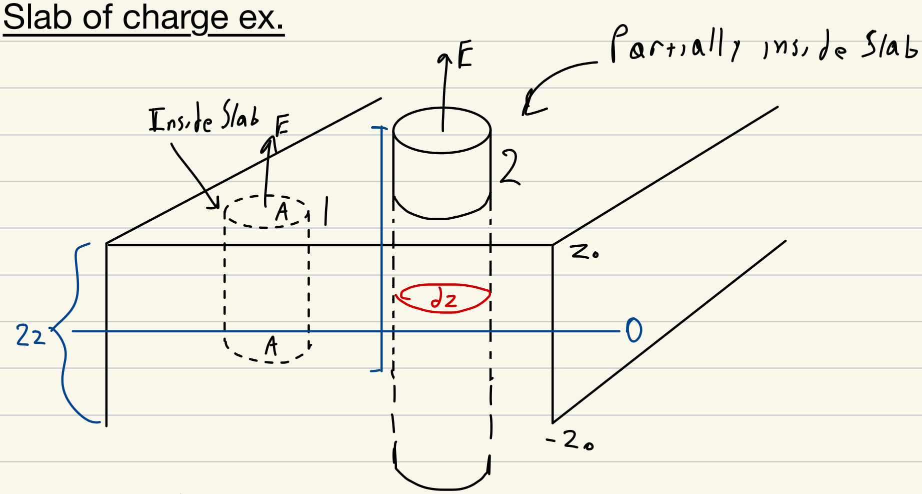
$$2EA = \frac{Q_{enc}}{\varepsilon_0}$$

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$\Delta V = E(y_2 - y_1)$$

Slab of charge ex.



1) $\oiint \vec{E}\, d\vec{A} = E_z 2A$

$\rho V_{enc} = Q_{enc}$

$\rho A \cdot 2z = Q_{enc}$

$E_z \cancel{2}\cancel{A} = \frac{\rho \cancel{A}\cancel{2}z}{\epsilon_0}$

$E(z) = \frac{\rho z}{\epsilon_0}$

---

2) $z > z_0$

$E(z)\cancel{2z_0} = \frac{\rho A \cancel{2z_0}}{\epsilon_0}$

$E(z) = \frac{\rho A}{\epsilon_0}$

$\rho = Cz^2 + B$

$Q_{enc} = \int \rho\, dV$

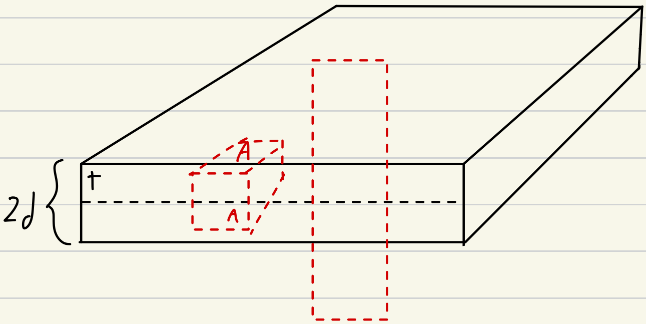
$dV = A\, dz$

$z < z_0)$ $\qquad$ $z > z_0)$

$Q_{enc} = \int_{-z}^{z} (Cz^2 + B) A\, dz \qquad = \int_{-z_0}^{z}$

$E(z) = \frac{Cz^3}{3\epsilon_0} + \frac{Bz}{\epsilon_0}$

Non-uniform Infinite Slab of Charge



$\rho(z) = Cz^2$

$|z| < d$

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$2EA = \frac{\int \rho \, dv}{\epsilon_0}$$

$$dv = A\,dz$$

$$\therefore \cancel{2}E\cancel{A} = \frac{\cancel{2}}{\epsilon_0}\int_0^z C z^2 \cancel{A}\, dz$$

$$E = \frac{Cz^3}{3\epsilon_0}$$

$|z| > d$

$$\cancel{2}E\cancel{A} = \frac{\cancel{2}}{\epsilon_0}\int_0^d C z^2 \cancel{A}\, dz$$

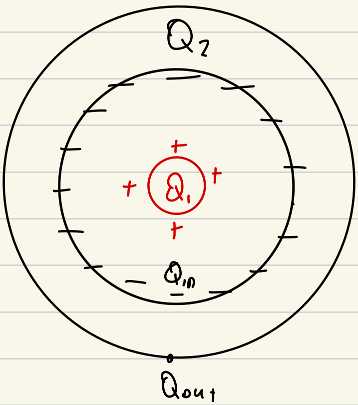
$$E = \frac{Cd^3}{3\epsilon_0}$$

$\Delta V;\ z_2 > d,\ z_1 < d \Rightarrow z_2 \rightarrow z_1$

$$\Delta V = -\int_{z_2}^{d} \vec{E} \cdot d\vec{z} + -\int_{d}^{z_1} \vec{E} \cdot d\vec{z}$$

$$= -\frac{Cd^3}{3\epsilon_0}(d - z_2) - \frac{C}{3\epsilon_0}\int_d^{z_1} z^3\, dz = -\frac{C}{3\epsilon_0}\left(\frac{3}{4}d^4 + \frac{z_1^4}{4} - z_2 d^3\right)$$

Metallic Cylinder with Cylinder of charge in Center Ex.



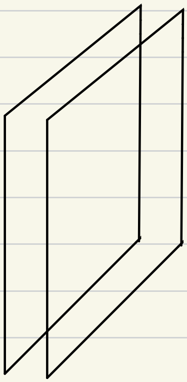
Metallic ∴ Conductor ∴ $\sum E = 0$

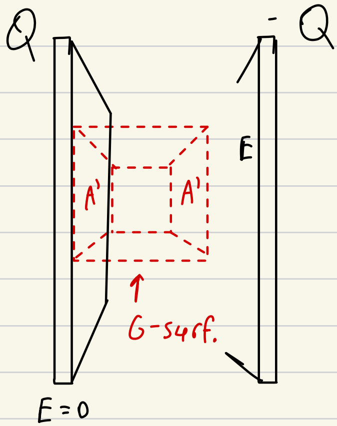
$$Q_{out} + Q_{in} = Q_2$$

$$Q_{out} - Q_1 = Q_2$$

$$Q_{out} = Q_1 + Q_2$$

Parallel Plate Capacitor (Conducting Plates):





$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

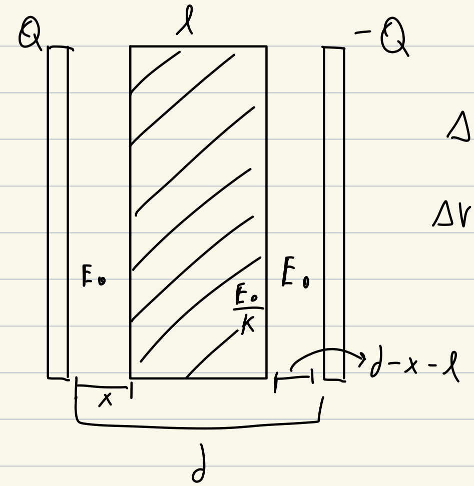
$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}d}{A\epsilon_0}} = \frac{A\epsilon_0}{d}$$

$$E\cancel{A'} + 0A' = \frac{\sigma \cancel{A'}}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

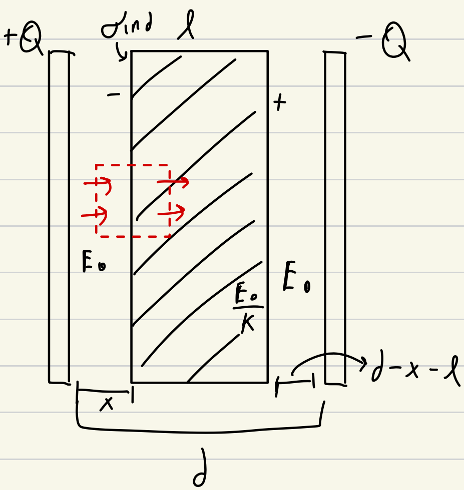
$$\Delta V = E d = \frac{Qd}{A\epsilon_0}$$

Parallel Plate Capacitor With Dielectric Slab:



$$\Delta V = E_0 x + \frac{E_0}{k} l + E_0 (d - x - l)$$

$$\Delta V = E_0 d + l\left(\frac{E_0}{k} - E_0\right)$$



$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E_0 A \cos\pi + \frac{E_0}{k} A' = \frac{Q_{enc}}{\epsilon_0}$$

$$-E_0 \cancel{A'} + \frac{E_0 \cancel{A'}}{k} = \frac{\sigma_{ind} \cancel{A'}}{\epsilon_0}$$

$$\sigma_{ind} = \epsilon_0 E_0 \left(\frac{1}{k} - 1\right)$$

If dielectric is metallic, k is infinity and 1/k drops

If the dielectric is metal it essentially acts as another capacitor

Why does sigma induced have to be less than that of plates?

$$E_{ind} = E_0 - \frac{E_0}{k}$$