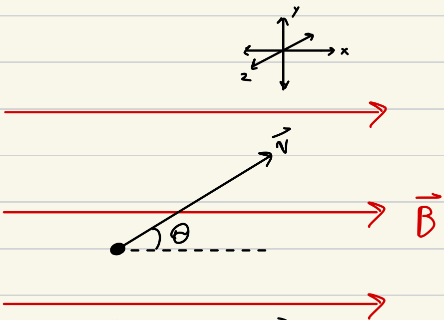
# Magnetism Intro

Magnetic Field:
- Caused by charges in MOTION (has to be charged)
- Electric field caused by charges that are stationary

Magnetic Domains:
- Magnets have North and South Pole
- You cannot separate poles
- Magnetic domains within materials
  - Magnetic atoms arranges/oriented in certain ways
  - Material composed of magnetic domains which are oriented either North-South, or other way around

- Diomagnetic
  - Polarized in reverse direction
- Paramagnetic:
  - Can be magnetized weakly
- Ferromagnetic:
  - Can be magnetized strongly




$\theta = \angle$ between $\vec{v}$ & $\vec{B}$

- Magnetic Field
  - Measured in Teslas

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$|\vec{F}_B| = |q|\,|\vec{v}|\,|\vec{B}| \sin\theta$$

Magnetic field goes backward on z-axis, perpendicular to velocity

Fb acts as a centripetal force

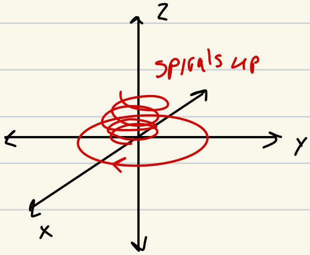
- When perpendicular to velocity

$$F_B = F_c$$
$$qvB = \frac{mv^2}{r}$$
$$r = mv/qB$$

$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$
$$T = \frac{2\pi m \cancel{v}}{qB\cancel{v}}$$
$$T = \frac{2\pi m}{qB}$$

If velocity is NOT at right angle to magnetic field

- Particle will spiral up
- Helix shape
- Pitch of the helix = distance moved along the axis while completing a full circle




- Pitch of helix = velocity in the direction of the axis x Time period of revolution

$$= v_{\parallel} \cdot T = v_{\parallel} \cdot \frac{2\pi m}{qB}$$

$v_{\parallel}$ => Component of velocity parallel to magnetic field

## Path of a moving charge:

I) $\vec{v} \parallel \vec{B}$ (ex. $\theta = 0°, 180°$)

Particle will move in straight line, no acceleration

II) $\vec{v} \perp \vec{B}$

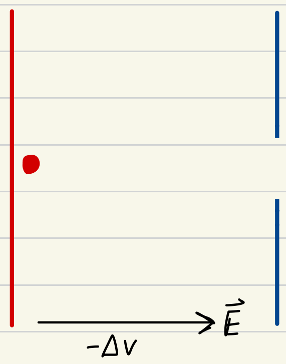
Particle will move in a circle, with r=mv/qB

III) $\vec{v}$ makes angle $\theta \neq 90, 0°, 180°$ with $\vec{B}$

Particle will move along a helical path with axis parallel to B

## Consider a Positive Ion (Mass Spectrometer):

I)




$q = 1e$

$\Delta U_E = -\Delta KE$

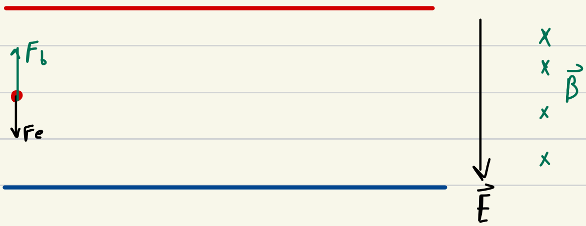
$q(\Delta V) = \cancel{\frac{1}{2}mv_i^2} - \frac{1}{2}mv_f$

$-q\Delta V = -\frac{1}{2}mv_f^2$

$v_f = \sqrt{\frac{2e\Delta V}{m}}$

Accelerate the particle

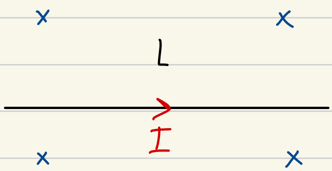
II)




$$\Sigma F = 0$$
$$\cancel{q} v B \sin\theta = \cancel{q} E$$
$$vB = E$$
$$v = E/B$$

Send accelerated particle through magnetic field and E field

Current through wire :

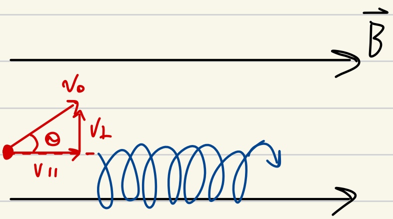



$$\vec{F_b} = I\vec{L} \times \vec{B}$$
$$= ILB\sin\theta$$

Derivation: $\vec{F_B} = q\vec{v} \times \vec{B}$
$$= q\left(\frac{\partial x}{\partial t}\right) \times B$$
$$= I L \times B$$

# The Helical Path

## Consider (B-Magnetic Field):




$$F_{B\perp} = q v_\perp \sin 90 = q v_\perp B$$

$$F_{B\parallel} = q v_{\parallel} \sin(0) = 0 \therefore v_{\parallel} \text{ constant}$$

$T = ?$

$$v_\perp = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v_\perp} \rightarrow F_B = m\frac{v^2}{r} \therefore r = \frac{m v_\perp}{qB}$$

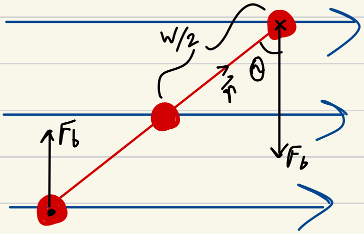
$$T = \frac{2\pi m}{qB}$$

$$\text{Pitch} = v_{\parallel} \cdot T$$

$$= v\cos\theta \cdot \frac{2\pi m}{qB}$$

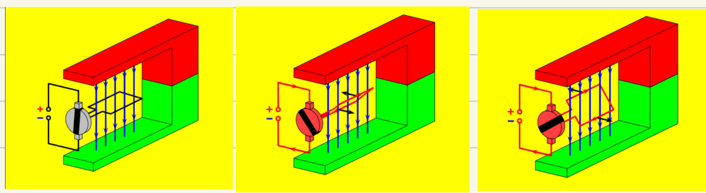
$$= \frac{2\pi m v \cos\theta}{qB}$$

# Rotating Motor with Magnetism




$\tau \rightarrow$ torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$



Side view of the rotating rectangle with current flowing through

$$|\vec{F}_B| = I\ell B \sin 90$$
$$= I\ell B$$

$$\vec{\tau} = \vec{r} \times \vec{F}_B$$
$$|\vec{\tau}| = \frac{W}{2} I\ell B \sin\theta$$

$$\sum\tau = -2\left(\frac{W}{2}\right) I\ell B \sin\theta$$
$$\sum\tau = -I\ell w B \sin\theta$$
$$\ell w = A$$

$$\downarrow$$

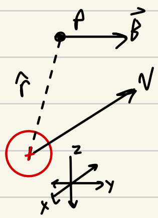
$$|\sum\tau| = IAB\sin\theta$$

$I\vec{A} = \vec{N} \rightarrow$ Magnetic moment of current charging coil

$$\boxed{\sum\tau = \vec{N} \times \vec{B}}$$

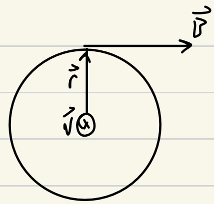
# The Magnetic Field

## Magnetic field at point P **PRODUCED** by particle :




$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{V}\times\hat{r}}{r^2} \qquad \hat{r} = \frac{\vec{r}}{|\vec{r}|} \quad \Rightarrow \frac{\mu_0}{4\pi}\frac{q\vec{V}\times\vec{r}}{r^3}$$

- This is for single charge




For ∞ Wire: $\vec{B} = \frac{\mu I}{2\pi r}$

- This is for a current

## Summary:

- To find magnetic field produced by particle at a point from the instantaneous position of particle:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{V}\times\vec{r}}{r^3}$$

$\mu_0$ = Permeability of free space

$= 4\pi \times 10^{-7}\ Tm/A$

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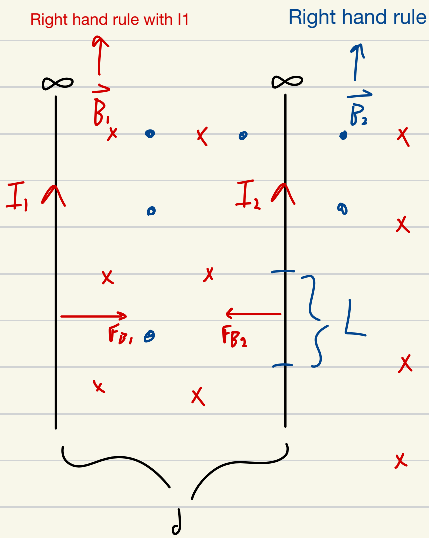


$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

$$F_{B2} = I_2 \vec{L} \times \vec{B}$$
$$= I_2 L B_1 \sin 90$$
$$= I_2 L \left(\frac{\mu_0 I_1}{2\pi d}\right)$$
$$F_{B_2} = I_2 L \mu_0 I_1 / 2\pi d$$

Force caused by B1 within/on segment L

$$\frac{F_{B_2}}{L} = \frac{I_2 I_1 \mu_0}{2\pi d}$$ → Force per unit length

$$F_{B_1} = I_1 \vec{L} \times \vec{B}_2$$
$$= I_1 L B_2 \sin 90$$
$$F_{B_1} = \frac{I_1 L \mu_0 I_2}{2\pi d}$$
$$\frac{F_{B_1}}{L} = \frac{I_1 I_2 \mu_0}{2\pi d}$$

Summary:

- Like **currents** (same direction) attract
- Opposite **currents** repel
- You can repeat last example for opposite direction currents

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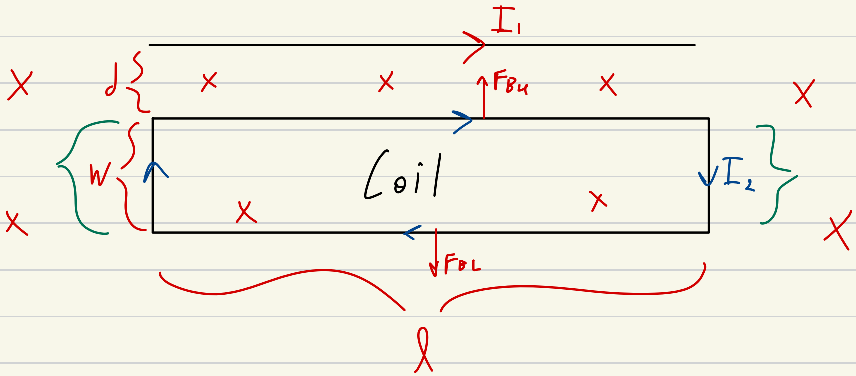



*Forces on right and left side cancel.*

- *Not uniform field on left and right cus it gets further; however, they cancel.*

$$\sum F_B = F_{Bu} - F_{BL} \qquad B_{1u} = \frac{\mu_0 I_1}{2\pi d}$$

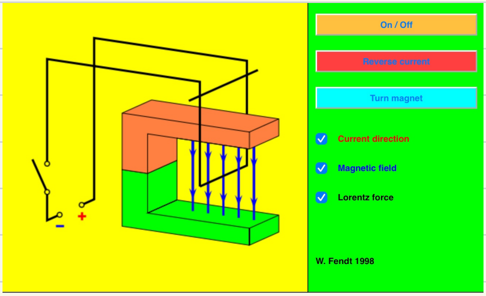
$$\sum F_B = \left(I_2 \vec{L} \times \vec{B_{1u}}\right) - \left(I_2 \vec{L} \times \vec{B_{1L}}\right) \qquad B_{1L} = \frac{\mu_0 I_1}{2\pi(d+w)}$$

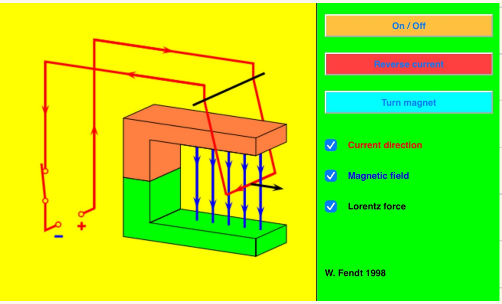
$$= \frac{I_2 \ell \mu_0 I_1}{2\pi d} - \frac{I_2 \ell \mu_0 I_1}{2\pi(d+w)}$$

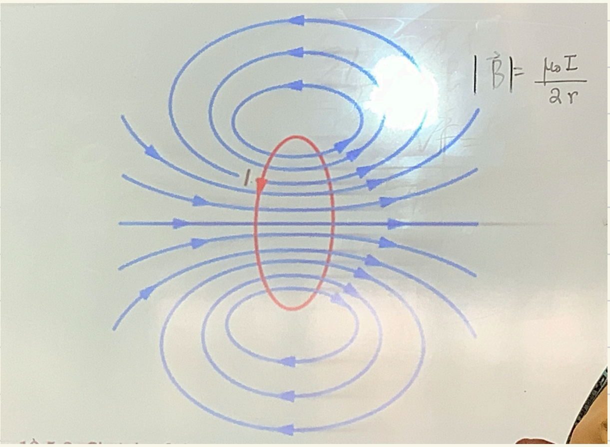
$$\boxed{\sum F_B = \frac{\mu_0 I_1 I_2 \ell w}{2\pi d(d+w)} \text{ upward}}$$

## Lorentz Force:

- Force exerted by magnetic field with current-carrying conductor



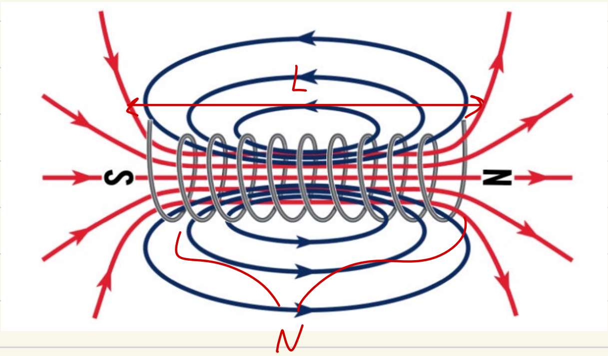




$$|\vec{B}| = \frac{\mu_0 I}{2r}$$

For Circular Current

<u>Solenoid:</u>

- Multiple circular currents




$\vec{B} = \mu_0 n I \rightarrow$ For solenoid

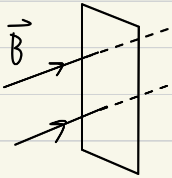
$n = \#\text{ of turns}/\text{length} = N/L$

$\vec{B} = \mu_0 N/L \, I$

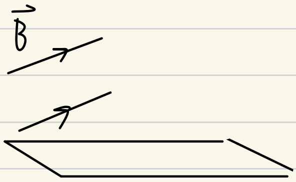
# Magnetic Induction

Magnetic Flux:

- How many field lines pierce through an area




Has flux




No flux

- Flux: $\Phi_B = |\vec{B}||\vec{A}|\cos\theta$

  $\theta = \angle$ between $\vec{B}$ & $\vec{A}$ (area vector)

  $\Phi_B = \vec{B} \cdot \vec{A}$ (Dot prod.)

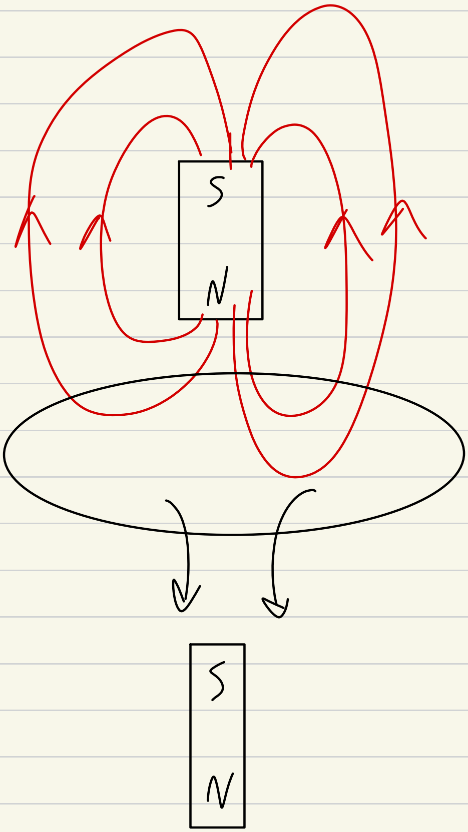
Faraday's Law:

- Given a closed conducting loop with magnetic field
- If the flux passed through it changes, then there will be an EMF generated
- Current will flow: EMF/r

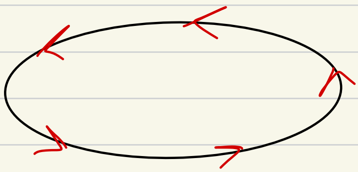
  $\mathcal{E} = -d\Phi_B/dt$

- Negative sign comes from Lenz's law
  - If the flux increases and causes EMF, then direction of EMF will be such that it tries to reduce the flux
  - Will also make opposing B-Field

Ex.

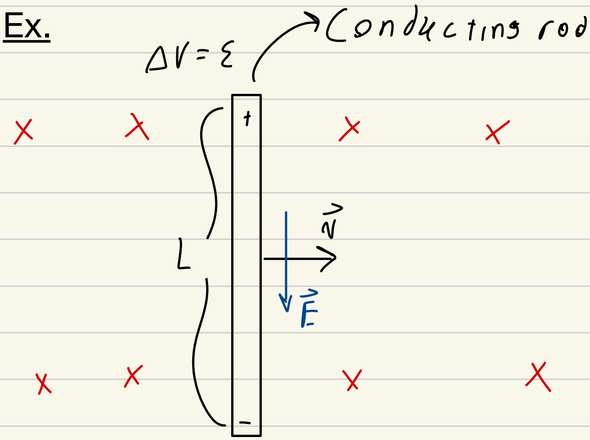



- As magnet approached coil, flux increases
- B-Field going up is made to oppose magnet B-field
- Current goes counter clockwise



- When magnet goes under coil, flux increases
- Current goes clockwise
- AC Current type thing

Ex.




- Magnetic force pushes electrons downwards, making a potential difference
- E Field Made
- Eventually, at equilibrium, Fe will equal Fb

$$E = \frac{\varepsilon}{L}$$

$$F_e = E\,e = e\frac{\varepsilon}{L}$$

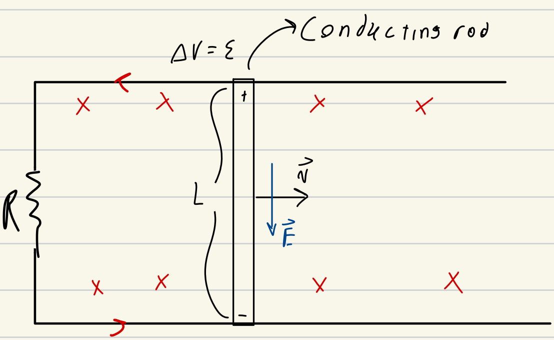
$$F_b = evB$$

$$F_B = F_e$$

$$\cancel{e}\frac{\varepsilon}{L} = \cancel{e}vB$$

$$\varepsilon = BLv$$

If there was a wire for current:




$$I = \varepsilon/R = \frac{BLv}{R}$$

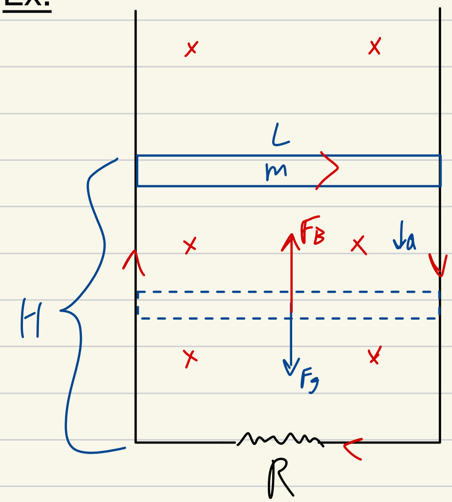
The current causes a magnetic field which causes an opposing force, slowing the rod down

$$\Phi_B = \vec{B} \cdot \vec{A} \rightarrow \text{Area } \vec{v} \text{ of circuit}$$
$$= |\vec{B}|\,|\vec{A}|$$

$$\mathcal{E} = \frac{-d\Phi_B}{dt} = \frac{-d}{dt}\left(|\vec{B}|\,|\vec{A}|\right) = -|\vec{B}|\frac{d|\vec{A}|}{dt} = -|\vec{B}|\frac{d}{dt}(dw) = -|\vec{B}|\,d\frac{dw}{dt} = -|\vec{B}|\,d\,v$$

$$\boxed{|\mathcal{E}| = |\vec{B}|\,d\,v}$$

Ex.




1) Rod starts falling due to gravity
2) Change in flux will happen and an opposing magnetic force will be applied

Sequence of Reasoning:
1) Magnetic field pointing into the board
2) Flux is changing because area changes. In this case, area decreases so flux decreases
3) Flux decreases, and so the EMF wants to support the magnetic field

$$\varepsilon(t) = BLv(t)\sin\theta$$

$$I(t) = \frac{BLv(t)\sin\theta}{R}$$

$$F_B = ILB$$

$$F_B = I(t)LB = \frac{B^2L^2v(t)\sin\theta}{R}$$

At terminal $v_T$

$$\Sigma F = 0$$

$$mg = \frac{B^2L^2v_T\sin\theta}{R}$$

$$v_T = \frac{mgR}{B^2L^2}$$

$$\Sigma F = F_g - F_B = ma$$

$$mg - \frac{B^2L^2v}{R} = m\frac{dv}{dt}$$

At $t=0$, $v=0$

$$\frac{dv}{dt} = \frac{mg - \frac{B^2L^2v}{R}}{m}$$

$$dt\left(\frac{dv}{dt}\right) = \left(g - \frac{B^2L^2v}{Rm}\right)dt$$

$$dv = \left(g - \frac{B^2L^2v}{Rm}\right)dt$$

$$\frac{dv}{g - \frac{B^2L^2v}{Rm}} = dt$$

$$\frac{-du\left(\frac{mR}{B^2L^2}\right)}{u} = dt$$

$$\int \frac{1}{u}\,du = \frac{-B^2L^2}{mR}\int dt$$

$$\ln u = -\frac{B^2L^2}{mR}(t) + C$$

$$\ln\left(g - \frac{B^2L^2}{mR}v\right) = \frac{-B^2L^2t}{mR} + C$$

$$\ln g = C$$

$$e^{\left(\ln\left(g - \frac{B^2L^2v}{mR}\right)\right)} = e^{\left(\frac{-B^2L^2t}{mR} + \ln g\right)}$$

$$g - \frac{B^2L^2v}{mR} = e^{-\frac{B^2L^2t}{mR}}(g)$$

$$v = \frac{mgR}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{mR}t}\right)$$

$$v = v_T\left(1 - e^{-\frac{B^2L^2}{mR}t}\right)$$

u-sub.

$$u = g - \frac{B^2L^2v}{mR}$$

$$\frac{du}{dv} = 0 - \frac{B^2L^2}{mR}(1)$$

$$\left(\frac{du}{dv}\right)dv = \left(-\frac{B^2L^2}{mR}\right)dv$$

$$du = \left(-\frac{B^2L^2}{mR}\right)dv$$

$$dv = -du\left(mR/B^2L^2\right)$$

## Varying B Field:

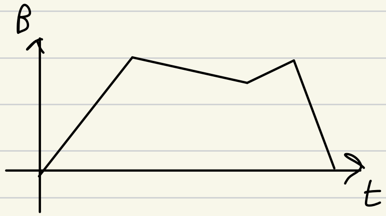
$$\Phi_B = BA\cos\theta$$

B varies w/ time

$$\mathcal{E} = \frac{d\Phi_B}{dt} = -A\cos\theta\left(\frac{dB}{dt}\right)$$

$$\mathcal{E} = -A\cos\theta\frac{d}{dt}\left(B_0 t^4\right)$$

$$= -AB_0\cos\theta\, t^3$$




## Varying Orientation:

$$\Phi_B = BA\cos\theta$$

$$\mathcal{E} = \frac{-d\Phi_B}{dt} = -BA\frac{d}{dt}\cos\theta$$

$$\mathcal{E} = -BA\frac{d}{d\theta}(\cos\theta)\frac{d\theta}{dt}$$

$$= -BA(-\sin\theta)\,d\theta/dt$$

$$\mathcal{E} = BA\sin\theta\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \omega$$

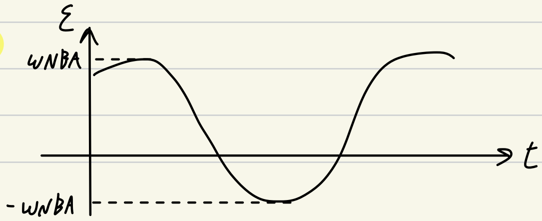
$$\int d\theta = \int\omega\, dt$$

$$\theta = \omega t + \phi$$

$$\mathcal{E} = \omega BA\sin(\omega t + \phi)$$

For coil w/ N turns

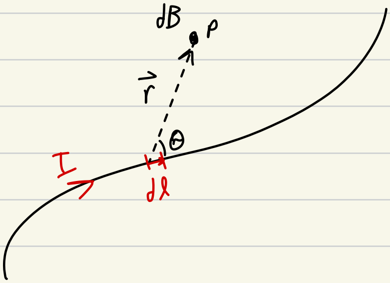
$$\mathcal{E} = \omega NBA\sin(\omega t + \phi)$$



# Magnetism

Biot-Savat Law:

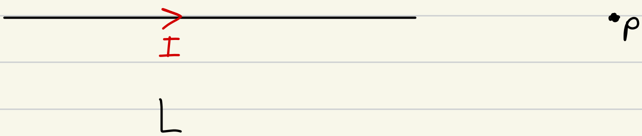
- B-Field due to various configs of current:



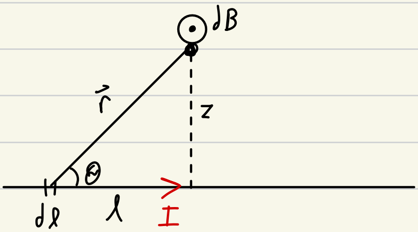

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\hat{r}}{r^2}$$

$$|\vec{dB}| = \frac{\mu_0}{4\pi}\frac{I\,dl\,r}{r^3}\sin\theta = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin\theta$$

Finite current element Ex.




$\vec{r} \parallel I \therefore \sin\theta = 0 \therefore B = 0$




L

$$|dB| = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin\theta$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\frac{z}{r}$$

$$B = \int \frac{\mu_0 I\,dl\,z}{4\pi(l^2+z^2)^{3/2}}$$

$$\cot\theta = \frac{l}{z}$$

$$l = z\cot\theta$$

$$dl = -z\csc^2\theta\,d\theta$$

$$B = \int \frac{\mu_0}{4\pi}\frac{I z(-\csc^2\theta)\,d\theta}{(z^2\cot^2\theta + z^2)^{3/2}}$$

$$= \int \frac{\mu_0 I z^2}{4\pi}\frac{-\csc^2\theta\,d\theta}{z^3(1+\cot^2\theta)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi z}\int \frac{-\csc^2\theta}{\csc^3\theta}\,d\theta$$

$$\Sigma B = 2\frac{-\mu_0 I}{4\pi z}\int \sin\theta\,d\theta$$

$$\frac{2\mu_0 I}{4\pi z}\cos\theta = \left[\frac{\mu_0 I}{2\pi z}\frac{l}{\sqrt{l^2+z^2}}\right]_0^{L/2}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi z}\left(\frac{L}{\sqrt{L^2/4+z^2}}\right)\ \text{out of board}}$$

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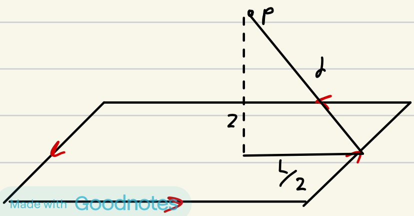
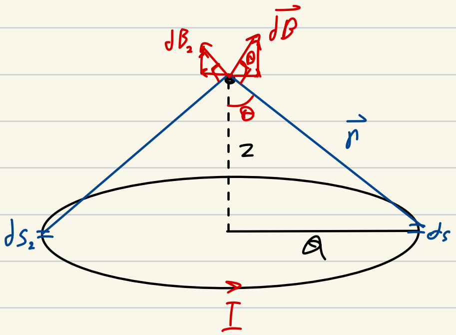



$$d = \sqrt{z^2 + \left(\frac{L}{2}\right)^2}$$

# Circular Loop Ex.




$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{ds}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I ds\, r \sin 90}{r^3}$$

$dB_x$ cancels out

$$dB_y = \frac{\mu_0}{4\pi}\frac{I ds}{r^2}\sin\theta = \frac{\mu_0}{4\pi}\frac{I ds}{r^2}\frac{a}{r} = \frac{\mu_0}{4\pi}\frac{I a}{r^3}ds$$

$$= \frac{\mu_0}{4\pi}\frac{I a}{(a^2+z^2)^{3/2}}ds$$

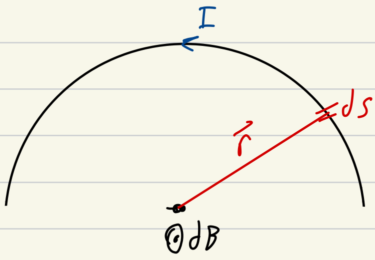
$$B = \int dB_y = \frac{\mu_0}{4\pi}\frac{I a}{(a^2+z^2)^{3/2}}\oint ds = \frac{\mu_0}{4\pi}\frac{I a}{(a^2+z^2)^{3/2}}2\pi a$$

$$B = \frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}$$

or

$$B = \int_0^{\frac{1}{2}s} 2\, dB_y$$

# Semi Circle Ex.




$$dB = \frac{\mu_0}{4\pi} \frac{I\,\vec{ds} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} \frac{I\, r \sin 90^\circ\, ds}{r^3}$$

$$B = \int_0^{S/2} \frac{\mu_0 I}{4\pi r^2}\, ds$$

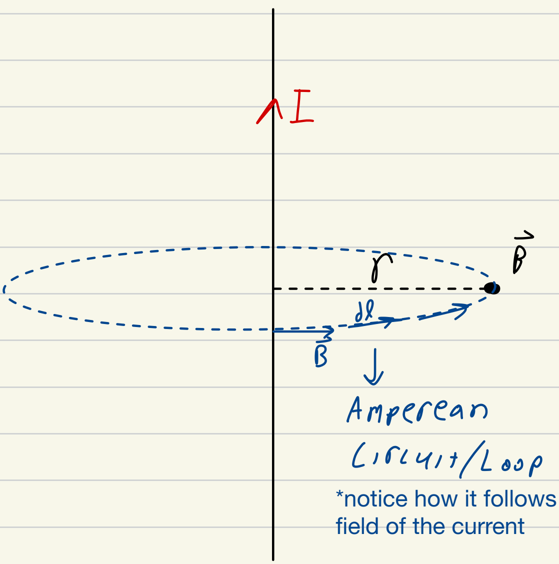
$$= \frac{\mu_0 I}{4\pi r^2}\, \pi r = \boxed{\frac{\mu_0 I}{4r}}$$

<u>Ampere's Law:</u>

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

- A good amperean path is a field line

Infinitely Long Current Carrying Conductor exam




*notice how it follows the B-field of the current

$$B \cdot \vec{dl} = B \cdot dl \cos(0) = B\,dl$$

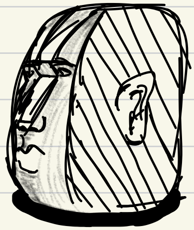
$$\oint \vec{B} \cdot dl = \vec{B} \oint dl = B 2\pi r$$

$$B 2\pi r = \mu_0 I_{enc} = \mu_0 I$$

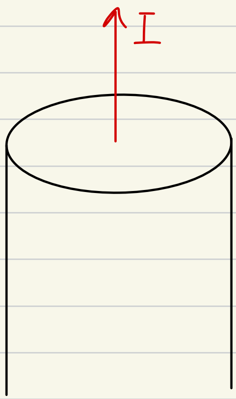
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

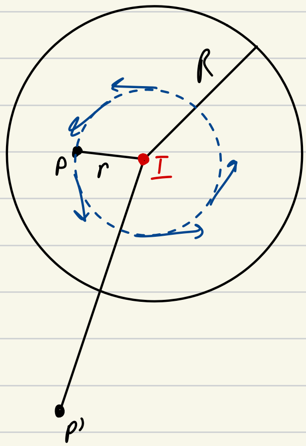
Current Density $\vec{j}$

$$I = \int \vec{j} \cdot d\vec{A}$$



Wire of Current Ex.




Uniform $\vec{j}$

Coming out of the screen

$r < R$)

$$\oint B \cdot dl = \oint B\,dl = B \oint dl = B 2\pi r$$

$$B 2\pi r = \mu_0 I_{enc}$$

$$B 2\pi r = \mu_0 \int \vec{j}\, d\vec{A} \cos(0)$$

$$B 2\pi r = \mu_0 j \int dA$$

$$B 2\cancel{\pi} r = \mu_0 j \cancel{\pi} r^2$$

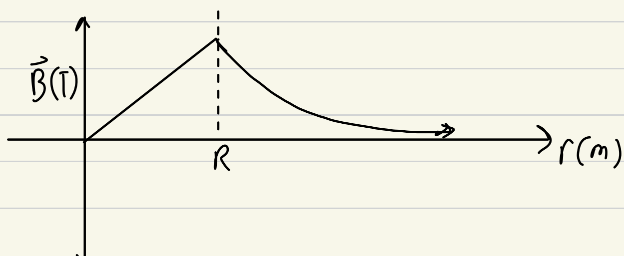
$$B = \frac{\mu_0 j r}{2}$$

$r > R$)

$$B 2\pi r = \mu_0 j \int dA$$

$$B 2\cancel{\pi} r = \mu_0 j \cancel{\pi} R^2$$

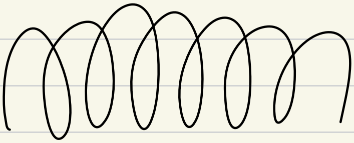
$$B = \frac{\mu_0 j R^2}{2r}$$

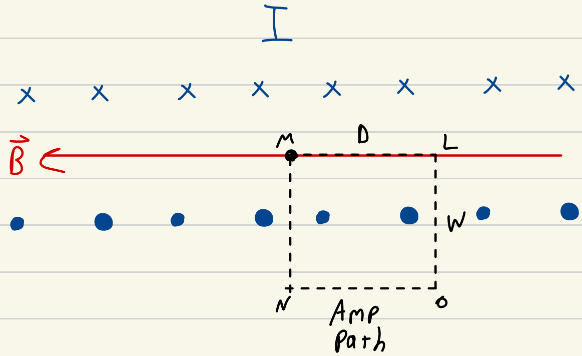



For non uniform:

$$I = \int j\,dA = \int j\, 2\pi r\, dr$$

# Solenoid Ex.






N turns, Length L

$$n = \frac{N}{L}$$

$$I_{enc} = nDI$$

$$\oint \vec{B}\cdot d\vec{l} = \underbrace{\oint B\cdot dl}_{LM} + \underbrace{\cancel{\oint B\cdot dl}}_{\substack{MN \\ B \perp dl}} + \underbrace{\cancel{\oint B\cdot dl}}_{\substack{NO \\ \vec{B}=0}} + \underbrace{\cancel{\oint B\cdot dl}}_{\substack{OL \\ B\perp dl}}$$

$$\oint B\cdot d\vec{l} = BD$$

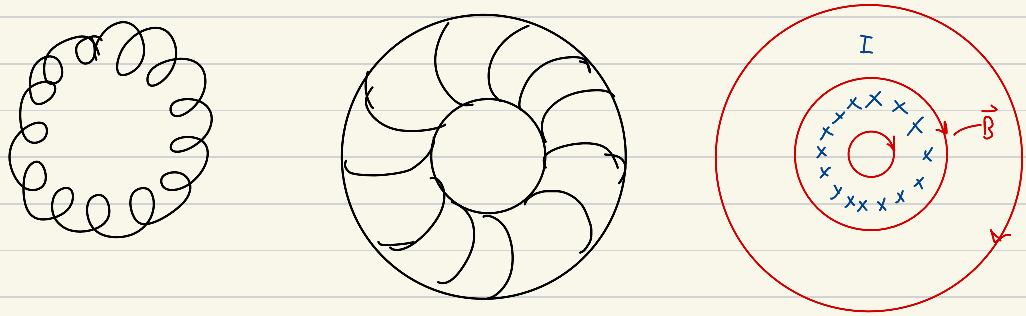
$$BD = \mu_0 I_{enc}$$

$$BD = \mu_0 \frac{N}{L} D I$$

$$B\cancel{D} = \frac{\mu_0 N \cancel{D} I}{L}$$

$$\boxed{B = \frac{\mu_0 N I}{L}}$$

Toroidal Solenoid Ex. (Solenoid turned into circle)




Inner Radius)
B = 0 b/c no enclosed current

$$\oint B \cdot dl = 0$$

$$B = 0$$

Outside Loop)
Also Solenoid because all currents cancel

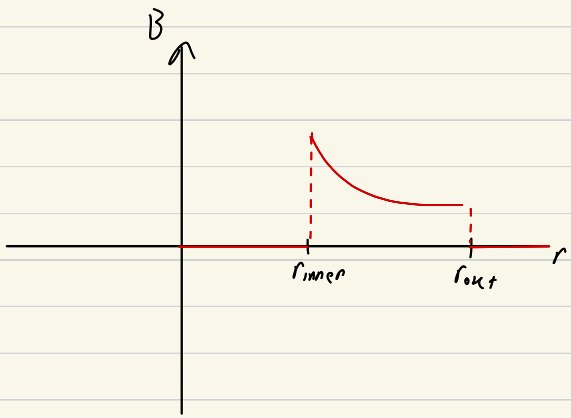
In between Area)

$$\oint B \cdot dl = \mu_0 I_{enc}$$

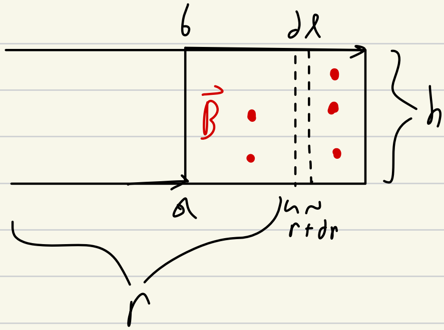
$$B \oint dl = \mu_0 I_{enc}$$

$$B 2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$



# Finding Flux Of Changing B-Field (toloid)




$$dA = h\,dl$$

Cross section of toroid

$$\iint \vec{B}\cdot d\vec{A} = \iint B(r)\,h\,dr = \frac{\mu_0 N I h}{2\pi}\int_a^b \frac{1}{r}\,dr = \frac{\mu_0 N I h}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\Phi_B = \frac{\mu_0 N I h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Gauss's Law:

Gaussian surface should be equipotential surfaces $\oiint \vec{E} \cdot \vec{dA}$

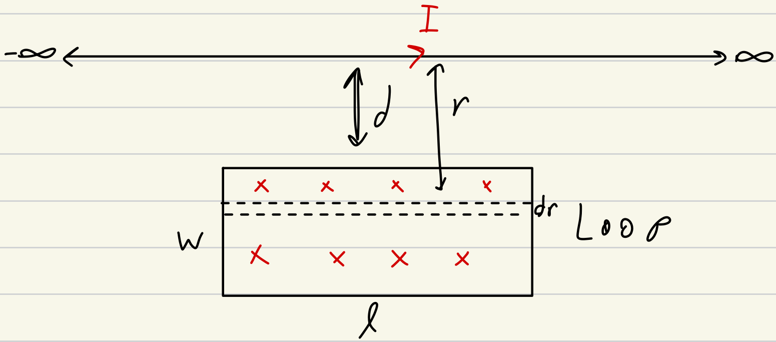
$\rho(\ ) \Rightarrow Q_{enc} = \int \rho \, dV$

Ampere's Law:

Amperean line should be B-field lines or perpendicular to it $\oint \vec{B} \cdot \vec{d\ell}$

$j(\ ) \Rightarrow I_{enc} = \iint \vec{j} \cdot \vec{ds}$

# Flux Ex.




$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B = \iint \vec{B}\cdot d\vec{A} = \int_{d}^{d+w} \frac{\mu_0 I}{2\pi r}\, l\, dr = \frac{\mu_0 I}{2\pi} l \ln\left(\frac{d+w}{d}\right)$$

# For Varying Current:

$$I = I_{max} \sin \omega t$$

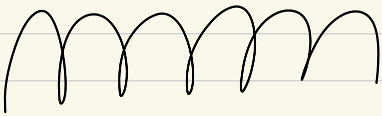
$$\Phi_B = \frac{\mu_0 l}{2\pi} I_{max} \sin \omega t \cdot \ln\left(\frac{d+w}{d}\right)$$

$$\varepsilon = \frac{-d\Phi_B}{dt} = \frac{\mu_0 I l w}{2\pi D(D+w)}$$

# Inductors

- Self Inductance: L (Measured in Henrys)

Inductance of Solonoid:



$l, N$

$$\Phi_B \propto I$$

$$\frac{d\Phi_B}{dt} \propto \frac{dI}{dt}$$

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

$$= -L\frac{dI}{dt}$$

$$\vec{B} = \frac{\mu_0 N I}{l}$$

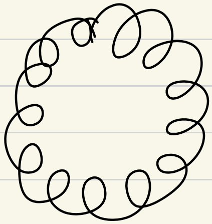
Let A be the cross-sectional area of a coil/turn

$$\Phi_B = \underbrace{NA}_{\Sigma A}(B) = N\frac{\mu_0 N I}{l}A = \frac{\mu_0 N^2 A}{l}I$$

$$\mathcal{E} = \frac{-d\Phi_B}{dt} = \frac{-\mu_0 N^2 A}{l}\frac{dI}{dt} \Rightarrow -L\frac{dI}{dt}$$

$$\therefore L = \frac{\mu_0 N^2 A}{l}$$

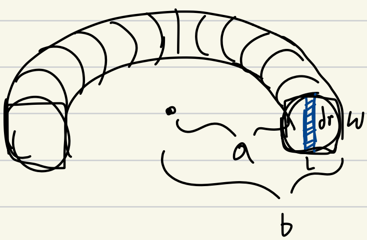
# Inductance of Toroidal Solonoid:



$$\mathcal{E} = \frac{-d\Phi}{dt} = -L\frac{dI}{dt}$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

Cross Section:




$$d\Phi_B = B\,dA = \frac{\mu_0 N I}{2\pi r}\,dr\,w$$

$$\Phi_B = \int_a^b \frac{\mu_0 N I}{2\pi r}\,w\,dr = \frac{\mu_0 N I w}{2\pi}\ln\left(\frac{b}{a}\right)$$

Flux for single turn

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$N\frac{-d\Phi}{dt} = -L\frac{dI}{dt}$$

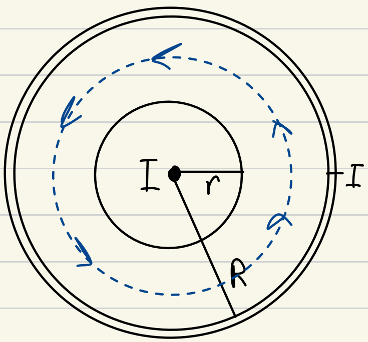
$$\frac{-\mu_0 N^2 w}{2\pi}\ln\left(\frac{b}{a}\right)\frac{dI}{dt}$$

$$L = \frac{\mu_0 N^2 w}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L = \frac{N\Phi_B}{I}$$

Made with Goodnotes

## Inductance of Co-Axial Cable(Opposite currents inner and outer circle):




Outside the circle (x>R)

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$\Sigma I_{enc} = 0 \therefore \vec{B} = 0$$

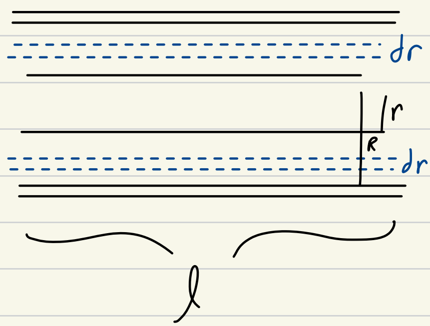
In Between)

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$B \, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Longitudal Slice




$$dA = l \, dr$$

$$d\Phi = B \, dA = \frac{\mu_0 I}{2\pi r} l \, dr$$

$$\Phi = \int_r^R \frac{\mu_0 I}{2\pi r} l \, dr = \frac{\mu_0 I}{2\pi} \ln\left(\frac{R}{r}\right)$$

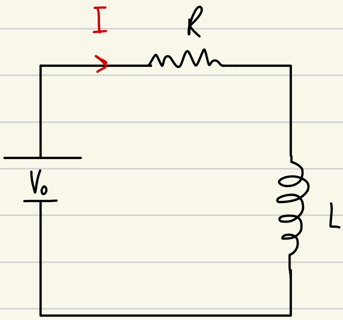
Inductance per Unit Length:

$$-\frac{d\Phi}{dt} = \frac{-\mu_0}{2\pi} \ln\left(\frac{R}{r}\right) \frac{dI}{dt} \therefore L = \frac{\mu_0}{2\pi} \ln\left(\frac{R}{r}\right)$$

# Inductor Circuits:

- Inductor has resistance

## Circuit Turned on:




Kirchoff's Rule)

$$V_0 + V_R + V_L = 0$$

$$V_0 - IR - L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = V_0 - IR$$

$$dI = \frac{1}{L}(V_0 - IR)\,dt$$

$$\frac{dI}{V_0 - IR} = \frac{1}{L}dt$$

$$u = V_0 - IR$$

$$du = -R\,dI$$

$$dI = \frac{-du}{R}$$

$$\frac{-du}{Ru} = \frac{1}{L}dt$$

$$\int \frac{du}{u} = \int -\frac{R}{L}\,dt$$

$$\ln u = \frac{-R}{L}t + C$$

$$\ln(V_0 - IR) = \frac{-Rt}{L} + C$$

$$t = 0,\ I = 0$$

$$\therefore C = \ln(V_0)$$

$$\ln(V_0 - IR) = \frac{-Rt}{L} + \ln(V_0)$$

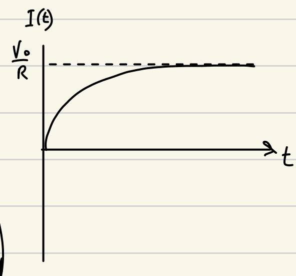
$$\ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{-R}{L}t$$

$$\frac{V_0 - IR}{V_0} = e^{-\frac{R}{L}t}$$

$$V_0 - IR = V_0 e^{-R/L\,t}$$

$$IR = V_0\left(1 - e^{-R/L\,t}\right)$$

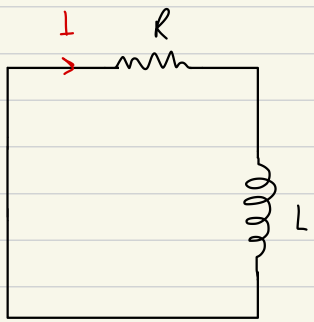
$$I = \frac{V_0}{R}\left(1 - e^{-R/L\,t}\right)$$




$\frac{V_0}{R}$ => steady state

When time reaches L/R (time constant), current reached 63.2%

Power Supply Removed (Circuit Turned off):




$V_0 = 0$

$\therefore -IR - L\frac{dI}{dt} = 0$

$-IR = L\frac{dI}{dt}$

$\frac{dI}{dt} = \frac{-R}{L} I$

$\frac{dI}{I} = -\frac{R}{L} dt$

$\ln I = \frac{-R}{L} t + C$

$t = 0, I = I_{max}$

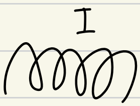
$\therefore C = \ln I_{max}$

$\ln\left(\frac{I}{I_{max}}\right) = -\frac{R}{L} t$

$$I = I_{max} e^{-\frac{R}{L} t}$$

- Inductors can store energy

$$E = \frac{1}{2} L I^2$$



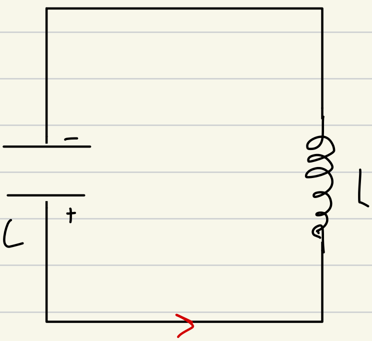

$$P = \mathcal{E} I = L \frac{dI}{dt} \cdot I$$

$$dE_L = P\,dt$$

$$E_L = L \int_0^{I(t)} I \frac{dI}{dt}\, dt$$

$$= L \int_0^{I(t)} I\, dI = \frac{1}{2} L I^2(t)$$

Circuit Ex (Ideal Inductor (no Res.)).




$t=0,\ Q_c = Q_0 \qquad I = \frac{-dQ}{dt}$

$V_C + V_L = 0$

$V_C = -V_L$

$\frac{Q}{C} = L\frac{dI}{dt} = L\frac{d}{dt}\left(\frac{-dQ}{dt}\right)$

$\frac{d^2Q}{dt^2} = -\frac{Q}{LC} \Rightarrow SHM$

$Q(t) = Q_0 \cos\left(\frac{t}{\sqrt{LC}}\right)$

Back and forth between magnetic energy and electric energy

If you Add resistance to inductor:

$L\frac{dI}{dt} + IR + \frac{Q}{C} = 0$

$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$

$\left(\frac{d^2}{dt^2} + \frac{R}{L}\frac{d}{dt} + \frac{1}{LC}\right)Q = 0$

$\frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$

**Displacement current?**